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Физика

Теоретический тур

Методическое пособие

Якутск, 1–8 июля 2008 г.

значение силы тока через резистор

$$I = I_e \cdot \frac{C_0}{C + C_0},$$

а потери энергии

$$\Delta W_T = I^2 RT = \frac{C_0^2 I_0^2}{2(C+C_0)^2} \cdot R \cdot 2\pi\sqrt{L(C+C_0)}.$$

Добротность

$$Q = \frac{2\pi W}{\Delta W_T} = \frac{L(C+C_0)^2}{C_0^2 R\sqrt{L(C+C_0)}} = \frac{1}{R}\left(\frac{C}{4\pi\varepsilon_0 r} + 1\right)^2 \sqrt{\frac{L}{C + 4\pi\varepsilon_0 r}}.$$

Примерная система оценивания

Замена сферы и заземления на эквивалентный конденсатор 1
Ответ для T 2
Ответ для $I_{\max}$ 3
Ответ для Q 4

Senior league

Problem 1. Oscillations on wires

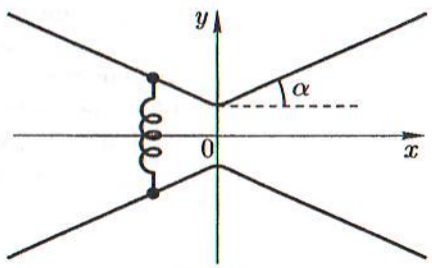

Fig. 13

Two identical wire angles are fixed in one plane in such a way that coordinate axes Ox and Oy are the axes of symmetry for them. (fig. 13). The angle between the sides of the angles and axis Ox is $\alpha = 30°$. Near the vertex of the angle the wire have a rounding that makes a smooth junction between the sides. Two identical beads can move along the wire angles. The coefficient of friction between the bead and the wire is $\mu = 0.1$. The beads are joined with a weightless spring which length in a free state is equal to the distance between the vertices of the angles.

1. Find the period T of oscillations at which the coordinates x of both beads are always equal.

2. Find the attenuation γ of such oscillations. Attenuation here means ratio of kinetic energy at passing position of equilibrium and kinetic energy after the expiration of a period.

The system is in a weightless state. If one end of the same spring is fixed and one of the beads is attached to the other end, the period of vertical oscillations of this pendulum is $T_0 = 1$ s.

Problem 2. Equinox days

Summer and winter solstice days (June, 21 and December, 21) divide year in two equal parts, and summer period between spring and autumn equinox days (from March, 20 to September, 22) is longer than winter period by $\tau \approx 7$ *days*.

1. When the distance between the Earth and the Sun is maximum and when it is minimum? Point out the dates, explain the answer.

2. Find the difference ΔR between maximum and minimum values of the distance between the Earth and the Sun.

3. Find the difference Δv between maximum and minimum values of orbit velocity of the Earth.

Note. Consider the Earth year to be $T \approx 365$ days; the Earth orbit differs a little from a circle with a radius $R = 1.5 \cdot 10^{11}$ m. At solstice days durations of day and night have the maximum difference and at equinox days they are equal. Due to the year's 2008 being a leap-year (366 days) the dates of solstice and equinox days differ from usual.

Problem 3. Refrigerator

A refrigerator has a lever regulating temperature in a chamber. The required temperature is maintained by means of special device, which turns the refrigerating system on and off periodically. When temperature in the chamber exceeds the required temperature by amount more than some fixed small value, this device turns the refrigerating system on. When temperature in the chamber gets lower than the required temperature by amount more than some fixed small value, this device turns the refrigerating system off.

In a steady state the temperature maintained in the chamber was $T_1 = 275$ K. The regulating lever had been turned, and after a while the temperature in the chamber became $T_2 = 270$ K.

1. How many times n the power of heat transfer from the environment to the chamber has changed?

2. How many times p the power of heat transfer from the chamber to the refrigerating system has changed during the period of operating of the system?

3. How many times α the percentage of the period of operating of the refrigerating system has changed?

4. How many times β the period of continuous operating of the refrigerating system has changed?

Represent the formulas and calculate numerical values accurate to the third sign after point.

The lowest temperature to be reached during continuous operating of the refrigerating system is $T_3 = 260$ K. The room temperature is $T_0 = 295$ K. The power the refrigerator takes from the electrical network during the period of operating of the refrigerating system is constant. The difference between temperatures in different parts of the chamber may be disregarded. Consider the coefficient of performance of the refrigerator to be some fixed number of times less than the coefficient of performance of an ideal cooler body operating accordingly to the Carnot cycle under the same conditions.

Note. The coefficient of performance is $\mu = Q/A$, where Q is the quantity of heat removed from the cooling body during the cycle, A is the work done on the body during the cycle.

Problem 4. Earthed circuit

An oscillatory circuit consisting of a coil with inductance L and a condenser with capacity C was earthed through a resistor with small resistance R (fig. 14). A conductive sphere with radius r is connected to the second terminal of the circuit. The condenser was charged up to the voltage U_0, then a key was closed. Immediately before the key was closed there were no oscillations in the circuit. The condenser and the sphere are far from each other.

1. Determine the period T of oscillations in the circuit.

2. Find the maximum current $I_{\max}$ in the coil during the oscillations.

3. Calculate the quality factor Q of the oscillating system.

Note. By definition quality factor is $Q = 2\pi W/\Delta W_T$, where W is full energy stored in a system, ΔW_T – losses of this energy over a period.

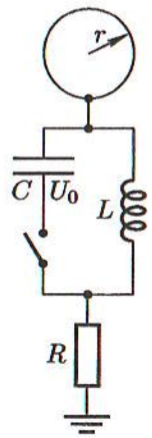

Fig. 14

Possible solutions

Senior league

Problem 1. Oscillations on wires

1. When a bead is at a distance l from an angle and move from it, it is affected with a force directed toward the angle

$$F_1 = F\sin\alpha + \mu N,$$

where $F = k\Delta y$ is the force of tension of the spring, $N = F\cos\alpha$ — the reaction of the wire, $\Delta y = 2l\sin\alpha$ — the amount by which the spring is stretched, whence

$$F_1 = 2\sin\alpha(\sin\alpha + \mu\cos\alpha)kl.$$

The force F_1 is proportional to the displacement l, that's why the effective deflection rate

$$k_1 = 2\sin\alpha(\sin\alpha + \mu\cos\alpha)k$$

may be introduced. By analogy we can find the force and the effective deflection rate when the bead moves toward the angle:

$$F_2 = F\sin\alpha - \mu N, \qquad k_2 = 2\sin\alpha(\sin\alpha - \mu\cos\alpha)k.$$

With the given values of α and μ the value $k_2 > 0$, hence the motion is oscillatory. During the full period of oscillation the beads move twice toward the angle and twice from the angle, with each of the four processes lasting a quarter of the corresponding period: $T_1 = 2\pi\sqrt{m/k_1}$ while moving from the angle and $T_2 = 2\pi\sqrt{m/k_2}$ while moving toward the angle. Thus, the total period is

$$T = 2\frac{T_1}{4} + 2\frac{T_2}{4} = \frac{T_0}{2\sqrt{\sin 2\alpha}}\left(\frac{1}{\sqrt{\operatorname{tg}\alpha + \mu}} + \frac{1}{\sqrt{\operatorname{tg}\alpha - \mu}}\right) \approx 1.43\ \text{s},$$

where we use the fact that $T_0 = 2\pi\sqrt{m/k}$. The time the beads need to pass the small rounding at the vertex of the angle may be disregarded.

2. Since the net force acting upon the bead is equivalent to the elastic force of some spring, energy losses when the beads move along the sides of the angle can be found by means of the abrupt changing of the effective deflection rate in the position of maximum deviation L. At this moment the full energy of the system equals the potential energy which decreases γ_1 times:

$$\gamma_1 = \frac{k_1L^2/2}{k_2L^2/2} = \frac{\operatorname{tg}\alpha + \mu}{\operatorname{tg}\alpha - \mu}.$$

Besides, there are energy losses when turning at the vertex of the angle. Let us put down the second Newton's law for the bead's deceleration on a small section of the wire with a radius of curvature R:

$$m\frac{dv}{dt} = -\mu m\frac{v^2}{R}.$$

If φ is the angular way of the bead then its increment is

$$d\varphi = |\omega|dt = \frac{v}{R}dt,$$

where ω is the angular velocity of the bead. Note that the modulus sign appears from definition of the angular way (in contradistinction to the displacement). Excluding R from the equations, we find

$$\frac{dv}{v} = -\mu d\varphi, \qquad \text{whence} \qquad v = v_0e^{-\mu\varphi}.$$

At the moment of passing the turning the full energy of the system equals the kinetic energy which decreases γ_2 times:

$$\gamma_2 = \left(e^{\mu\cdot 2\alpha}\right)^2 = e^{4\alpha\mu},$$

where we substitute $\varphi = 2\alpha$ and take into consideration that $E_{kin} \sim v^2$. During the full period of oscillation the beads are twice in the position of maximum distance and pass the turn twice, so the total attenuation is

$$\gamma = \gamma_1^2\gamma_2^2 = e^{8\alpha\mu}\left(\frac{\operatorname{tg}\alpha + \mu}{\operatorname{tg}\alpha - \mu}\right)^2 \approx 1.52\cdot 2.01 \approx 3.$$

Marking scheme

k_1 and k_2 were found 1
Formula for T 2
Numerical answer for T 1
γ_1 was found 2
γ_2 was found 2
Formula for γ 1
Numerical answer for γ 1

Problem 2. Equinox days

1. According to the first Kepler's law the Earth moves around the Sun along an ellipse with the Sun in one of the focuses F (fig. 15). Changing of duration of day and night during the year is provided by the fact that the axis of self-rotation of the Earth is not perpendicular to the plane of the orbit, but is inclined to it. According to the law of conservation of angular momentum this axis does not

change its direction. At solstice day its projection on the plane of the orbit of the Earth is parallel to the line connecting the Earth and the Sun, at equinox day — perpendicular to it. Hence, the Sun is both on the line connecting the points at which the Earth is at solstice days (points B and D) and on the line connecting the points at which the Earth is at equinox days (points A and C), the lines being perpendicular to each other. On the figure the projections of the axis of self-rotation of the Earth are shown with short lines.

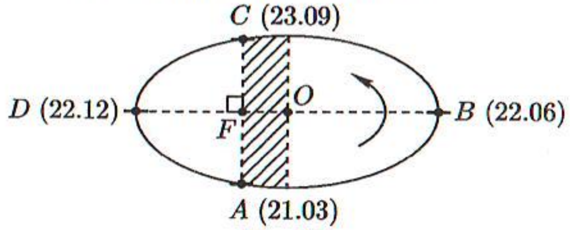

Fig. 15

The second Kepler's law (about constant areal velocity) and the fact that summer and winter solstice days divide year in two equal parts result in that: the line BD divide the ellipse in two parts with equal areas. This is possible only if this line contains the centre O of the ellipse, i.e., it is the major axis of the ellipse. Since summer period between spring and autumn equinox days is longer than winter one, an area of a segment ABC is larger that an area of a segment CDA due to the second Kepler's law, whence $BF > DF$. That's why June, 21 the Earth is at maximum distance from the Sun, December, 21 — at minimum distance.

2. Let S_B be the area of the segment ABC, T_B — the time the Earth needs to go along the path ABC, then due to the second Kepler's law

$$\frac{S_B}{T_B} = \frac{\pi R^2}{T}. \tag{13}$$

Considering that the Earth's orbit differs a little from a circle, one may treat the shaded area as a rectangle and put down

$$S_B \approx \frac{\pi R^2}{2} + 2Rf,$$

where $f = FO$ is a focal distance of the ellipse. From the condition $T_B + (T_B - \tau) = T$ we find $T_B = (T + \tau)/2$. Substituting the expressions for S_B and T_B в (13), we express

$$f = \frac{\pi\tau}{4T} R.$$

The required difference between the distances is

$$\Delta R = 2f = \frac{\pi\tau}{2T} R \approx 4{,}5 \cdot 10^9 \text{ m}.$$

3. Let v_B and v_D be correspondingly the velocities of the Earth at points B and D, $R_B = BF$, $R_D = DF$, then it follows from the second Kepler's law that $v_B R_B = v_D R_D$. The required difference between the velocities is

$$\Delta v = v_D - v_B = \frac{R_B - R_D}{R_B} \cdot v_D = \frac{\Delta R}{R_B} \cdot v_B \approx \frac{\Delta R}{R} \cdot v,$$

where $v = 2\pi R/T$ is the average orbit velocity of the Earth. Thus,

$$\Delta v = \frac{2\pi \Delta R}{T} = \frac{\pi^2 \tau R}{T^2} \approx 900 \text{ m/s}.$$

Note. It's interesting to point out that the eccentricity of the Earth orbit

$$\varepsilon = \frac{f}{R} = \frac{\pi\tau}{4T} \approx 1.5\%$$

can be calculated with good accuracy with only a calendar being used!

Marking scheme

Both of the dates (1 point for each) 2
Explanation of the dates 2
Formula for ΔR 2
Numerical answer for ΔR 1
Formula for Δv 2
Numerical answer for Δv 1

Problem 3. Refrigerator

All the values related to the steady state before turning the lever are marked with index "1", after it — with index "2".

1. According to the heat-conduction law the powers of heat transfer from the environment to the chamber can be presented as

$$N_1 = k(T_0 - T_1), \qquad N_2 = k(T_0 - T_2),$$

where k is the constant of proportionality depending only on size of the chamber and on properties of its walls. The required proportion is

$$n = \frac{N_2}{N_1} = \frac{T_0 - T_2}{T_0 - T_1} = 1.250.$$

2. The coefficient of performance of an ideal cooler body operating accordingly to the Carnot cycle is $\mu = T_X/(T_O - T_X)$, where T_X and T_O are correspondingly the temperatures of the cooling body and of the environment.

Let P be the power the refrigerating system takes from the electrical network, η — the ratio of the efficiency of the refrigerator to the efficiency of the ideal cooling body, then the powers of heat transfer from the chamber to the refrigerating system can be expressed as

$$P_1 = \eta \cdot \frac{T_1}{T_0 - T_1} \cdot P, \qquad P_2 = \eta \cdot \frac{T_2}{T_0 - T_2} \cdot P.$$

The required proportion is

$$p = \frac{P_2}{P_1} = \frac{T_0 - T_1}{T_0 - T_2} \cdot \frac{T_2}{T_1} \approx 0.785.$$

3. Let t_1 and t_2 be the periods of continuous operating of the refrigerating system, τ_1 and τ_2 — the periods between two successive turnings on or turnings off, then equations of thermal balance take the following form:

$$P_1 t_1 = N_1 \tau_1, \qquad P_1 t_1 = N_1 \tau_1,$$

whence the percentages of the periods of operating of the refrigerating system are

$$\alpha_1 = \frac{t_1}{\tau_1} = \frac{N_1}{P_1} = \frac{k(T_0 - T_1)^2}{\eta P T_1}, \qquad \alpha_2 = \frac{t_2}{\tau_2} = \frac{N_2}{P_2} = \frac{k(T_0 - T_2)^2}{\eta P T_2}.$$

The required proportion is

$$\alpha = \frac{\alpha_2}{\alpha_1} = \frac{(T_0 - T_2)^2}{(T_0 - T_1)^2} \cdot \frac{T_1}{T_2} \approx 1.591.$$

4. When the refrigerating system operates continuously the power of heat transfer from the environment to the chamber is equal to the power of heat transfer from the chamber to the refrigerating system:

$$k(T_0 - T_3) = \eta \cdot \frac{T_3}{T_0 - T_3} \cdot P, \qquad \text{whence} \qquad \frac{\eta P}{k} = \frac{(T_0 - T_3)^2}{T_3}.$$

Since the heat capacity of the chamber and permissible deviations from the required temperature are constant, during the period of continuous operating of the refrigerating system the same total quantity of heat Q is removing from the chamber:

$$Q = (P_1 - N_1)t_1, \qquad Q = (P_2 - N_2)t_2,$$

whence the required proportion is

$$\beta = \frac{t_2}{t_1} = \frac{P_1 - N_1}{P_2 - N_2} = \frac{\dfrac{\eta P T_1}{T_0 - T_1} - k(T_0 - T_1)}{\dfrac{\eta P T_2}{T_0 - T_2} - k(T_0 - T_2)} =$$

$$= \frac{\left(T_1(T_0 - T_3)^2 - T_3(T_0 - T_1)^2\right)(T_0 - T_2)}{\left(T_2(T_0 - T_3)^2 - T_3(T_0 - T_2)^2\right)(T_0 - T_1)} \approx 1.730.$$

Marking scheme

Formula and numerical answer for n 1
Formula and numerical answer for p 1
Formula for α 2
Numerical answer for α 1
Formula for β 4
Numerical answer for β 1

Problem 4. Earthed circuit

Let us treat the conducting sphere and the Earth as two plates of a condenser with some capacity C_0 and draw an equivalent circuit (fig. 16). Imagine a charge q put on the sphere, then the potential difference between the sphere and an infinitely distant point is $U = q/(4\pi\varepsilon_0 r)$, whence

$$C_0 = \frac{q}{U} = 4\pi\varepsilon_0 r.$$

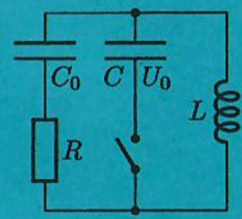

Fig. 16

Immediately before the key was closed the voltage on the condenser C_0 was equal to zero because there were no oscillations in the circuit.

1. Since the resistance R is small, the condensers may be treated as multipled, then

$$T = 2\pi\sqrt{L(C + C_0)} = 2\pi\sqrt{L(C + 4\pi\varepsilon_0 r)}.$$

2. Immediately after the key was closed the initial charge $q_0 = CU_0$ is distributed over both condensers, therefore voltage on each of them is

$$U_1 = \frac{q_0}{C + C_0}.$$

Since R is small, that occurs in a period much less than the period of oscillations and oscillations may be considered to begin only after charge distribution. Note that by virtue of a great current flowing through the resistor sufficient quantity of heat is generated during this small period, therefore it is a mistake to consider initial energy of oscillations to equal $CU_0^2/2$.

But heat generation during the first quarter of the oscillating period may be disregarded, then from the energy conservation law

$$\frac{(C + C_0)U_1^2}{2} = \frac{LI_{\max}^2}{2}$$

we can find

$$I_{\max} = U_1\sqrt{\frac{C + C_0}{L}} = \frac{CU_0}{\sqrt{L(C + 4\pi\varepsilon_0 r)}}.$$

Later current diminishes because of energy losses on the resistor.

3. Let I_0 be a current flowing through the coil at a moment when the condensers are discharged, then energy stored in the circuit is $W = LI_0^2/2$, root-mean-square value of current during the next oscillating period is $I_e = I_0/\sqrt{2}$. The currents flowing through the condensers are proportional to their capacities because the resistance R is small and the voltage drop on it may be neglected. Therefore, during the oscillating period the root-mean-square value of a current flowing through the resistor is

$$I = I_e \cdot \frac{C_0}{C + C_0},$$

energy losses are

$$\Delta W_T = I^2 RT = \frac{C_0^2 I_0^2}{2(C + C_0)^2} \cdot R \cdot 2\pi\sqrt{L(C + C_0)}.$$

The quality factor is

$$Q = \frac{2\pi W}{\Delta W_T} = \frac{L(C + C_0)^2}{C_0^2 R\sqrt{L(C + C_0)}} = \frac{1}{R}\left(\frac{C}{4\pi\varepsilon_0 r} + 1\right)^2 \sqrt{\frac{L}{C + 4\pi\varepsilon_0 r}}.$$

Marking scheme

The sphere and the Earth were replaced with the equivalent condenser 1
Answer for T 2
Answer for $I_{\max}$ 3
Answer for Q 4