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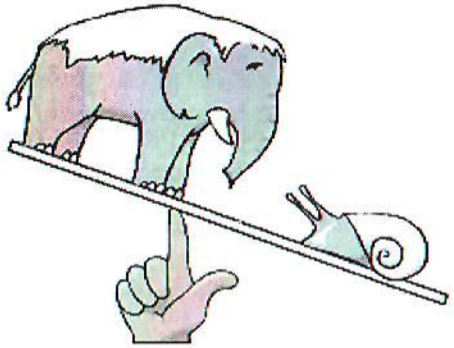
Физика

Экспериментальный тур

Методическое пособие



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между местами крепления вертикальной планки маятника: для этого достаточно с помощью отвертки немного ослабить саморезы и, установив необходимую длину планки, вновь закрепить её. Нить необходима для гашения мод колебаний маятника, не связанных с кручением.

Задание

1. Используя стробоскопический тахометр, снимите зависимость периода $T(\ell)$ крутильных колебаний Т-образного маятника от расстояния ℓ между креплениями вертикальной планки. Аналогичные измерения проведите, поменяв в Т-образной конструкции планки местами.
2. Получите формулу для периода крутильных колебаний Т-образного маятника, выразив его через момент инерции и модуль кручения маятника.
3. Используя общефизические соображения, получите (теоретически) явную зависимость периода $T(\ell)$ крутильных колебаний маятника от ℓ.
4. Сравните экспериментальную зависимость $T(\ell)$ с теоретической, полученной в п.3. Сделайте вывод о значении показателя степени q в формуле для модуля кручения $\kappa = \beta G^m \delta^n b^p \ell^q$.
5. Используя экспериментальные результаты, выясните зависимость модуля кручения κ от ширины линейки b, определив показатель степени p в формуле для модуля кручения $\kappa = \beta G^m \delta^n b^p \ell^q$. При выполнении этого задания считайте, что момент инерции маятника определяется только моментом инерции поперечной алюминиевой планки. Обоснуйте это предположение, сравнив момент инерции планки с моментом инерции крепежа (деревянной линейки и крепёжных клипс) .
6. Определите (теоретически) показатели степени m и n в формуле для модуля кручения, получив, таким образом, явную зависимость модуля кручения от модуля сдвига G и толщины планки δ. Запишите окончательное выражение для модуля кручения κ, положив безразмерный коэффициент $\beta = 1/3$.
7. Придумайте и проведите эксперимент, позволяющий с помощью имеющегося оборудования определить численное значение модуля кручения κ для одной из алюминиевых планок.
8. Рассчитайте модуль сдвига G алюминия.

Modulus of torsion of ruler

Necessary information

Torsional deformation.

We will turn a free end of metal ruler (a lath) relative to the other fixed end by an angle φ around a longitudinal axis of symmetry of ruler, as is shown in figure 1. At the same time different sections of ruler are rotated relative to the fixed end at different angles. Such a deformation is called ***torsion***. For small deformations according to Hooke's law the torque M required for turning the ruler is proportional to the angle φ:

$$M = k\varphi .$$

The coefficient of proportionality κ is called a ***modulus of torsion*** of ruler.

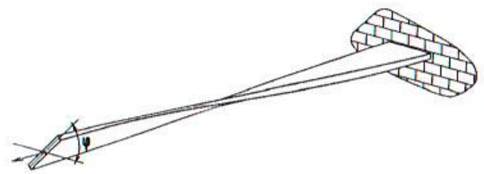


Fig. 1

The modulus of torsion determines geometric parameters of ruler and elastic properties of material from which the ruler was made. So, for the thin ruler of length l, width b and thickness δ made from a material with a ***shear modulus*** G, the modulus of torsion can be calculated according to a formula:

$$k = \beta G^m \delta^n b^p l^q ,$$

where m, n, p, q are some integers, and β is a dimensionless coefficient.

Shear modulus, shear deformation.

Along with Young's modulus, the shear modulus G is the most important constant characterizing the elastic properties of homogeneous isotropic material. It is determined from Hooke's law for ***shear deformation***. The shear deformation occurs when a force F is applied to a bar not perpendicularly, but tangentially to its surface. So, if a bottom side of bar is fixed and a tangential force F is applied to the top side of area S, then the parallelepiped will be twisted, as is shown in figure 2.

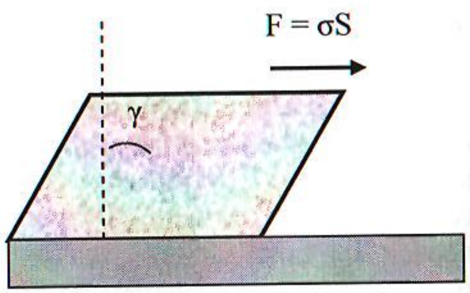


Рис. 2

For small deformations a skew angle γ is related with a magnitude of tangential stress $\sigma = F/S$ by a relation:

$$\gamma = \frac{\sigma}{G}.$$

This is Hooke's law for the shear deformation.

Purpose of work.

In this work the elastic properties of aluminium laths (rulers) are studied with the use of torsional oscillations.

In the work it is required:

- being guided by physical considerations, to determine exponents m, n, p and q in the formula for the modulus of torsion κ of ruler by a method of dimensions and having carried out necessary measurements;
- to experimentally determine a <u>numerical</u> value of modulus of torsion κ of one of rulers;
- to determine the shear modulus G of material, from which the rulers were made, with the use of known dimensionless coefficient $\beta = \frac{1}{3}$.

Equipment. Two aluminium laths (rulers) with the same thickness $\delta = 2$ mm, a short wooden ruler, a wooden bar with a fixed metal lath, two clamps, stationery clips (clamps), 4 identical magnets each having a mass m = 3.75 g, a measuring ruler (a paper meter), a thread, a stroboscopic tachometer, a crosshead screwdriver, a felt-tip pen for marks on aluminium strips.

Note: *the aluminium laths with the use of short wooden ruler and stationery clips are combined in a T-shaped structure, which is a torsional pendulum. The structure of pendulum allows regulating a distance l between places of fixation of vertical lath of pendulum: for this it is enough with the use of screwdriver to weaken a little bit crews and, after determining a necessary length of lath, to fix it again. The thread is necessary for damping oscillation modes of pendulum, not associated with torsion.*

Task

1. Using the stroboscopic tachometer, to obtain a dependence of period $T(l)$ of torsional oscillations of T-shaped pendulum on the length l between fixations of vertical lath (**3**). Carry out similar measurements, changing the places of laths in the T-shaped structure.
2. Get a formula for the period of torsional oscillations of T-shaped pendulum, expressing it through a moment of inertia and modulus of torsion of pendulum.
3. Using general physical considerations, obtain (theoretically) an explicit dependence of period $T(l)$ of torsional oscillations of pendulum on l.

4. Compare the experimental dependence $T(l)$ with the theoretical one obtained in point 3. Draw a conclusion about a value of exponent q in the formula for modulus of torsion $k = \beta G^m \delta^n b^p l^q$
5. Using the experimental results, find out a dependence of modulus of torsion, κ on a width of ruler b, determining the exponent p in the formula for modulus of torsion $k = \beta G^m \delta^n b^p l^q$. When performing this task you may assume that the moment of inertia of pendulum is determined only by the moment of inertia of transversal aluminium lath. Justify this assumption by comparing the moment of inertia of lath with the moment of inertia of fixation (the wood ruler and fastening clips) **(6)**.
6. Determine (theoretically) the exponents m and n in the formula for the modulus of torsion, thus having obtained an explicit dependence of module of torsion on the shear modulus G and thickness of lath δ. Write down the final expression for the modulus of torsion κ, putting a dimensionless coefficient $\beta = \frac{1}{3}$.
7. Design and conduct an experiment that enables with the use of existing equipment to determine the numerical value of modulus of torsion κ for one of aluminium laths. **(4)**
8. Calculate the shear modulus G of aluminium.

Modulus of torsion of ruler
Possible solution.

1. *Dependence T(l).*

By varying the length l of ruler fixed in the tripod that defines the elasticity of torsional pendulum, with the help of tachometer we obtain the dependence $v(l)$ of oscillation frequency v of pendulum on l.

- First, the ruler of width $b_1 = 20\ mm$ is fixed in the tripod; a width of transverse ruler is $b_2 = 15\ mm$. Results of measurements $v(l)$ are presented in Table 1.
- Interchange the places of rulers: the width of fixed ruler $b_2 = 15\ mm$, the width of transverse ruler $b_1 = 20\ mm$. Results $v(l)$ for this pendulum are in Table 2.

Table 1

№	Width of fixed ruler b_1 = 20 mm			
	l, mm	v, min^{-1}	T, s	T^2, s^2
1	400	560	0.107	0.0115
2	350	599	0.100	0.0100
3	300	637.5	0.094	0.00886
4	250	691.5	0.087	0.00753
6	200	760	0.079	0.00623
6	150	850	0.0705	0.00498
7	100	985	0.0609	0.00371
	$T^2(s^2) = B_1 l = 2{,}96\cdot10^{-5} l\ (mm)$			

Table 1

№	Width of fixed ruler b_2 = 15 mm			
	l, mm	v, min^{-1}	T, s	T^2, s^2
1	400	427.8	0.140	0.0197
2	350	452.5	0.133	0.0176
3	300	482.5	0.124	0.0155
4	250	523	0.115	0.0132
6	200	582.5	0.103	0.0106
6	150	655.5	0.0915	0.00838
7	100	771	0.0778	0.00606
	$T^2(c^2) = B_2 l = 5{,}11\cdot10^{-5} l\ (mm)$			

2. ***Period of oscillation of torsional pendulum.***

An equation of harmonic oscillations:

$$I\varphi'' + k\varphi = 0. \tag{1}$$

A cyclic frequency:

$$\omega_o^2 = \frac{k}{I} \tag{2}$$

A period of oscillations:

$$T = 2\pi\left(\frac{I}{k}\right)^{1/2} \tag{3}$$

Here I is the moment of inertia of pendulum, practically equal to the moment of inertia of transverse ruler relative to an axis of rotation of pendulum

$$I \approx \frac{Ml_0^2}{12}, \tag{4}$$

where M is the mass of transverse ruler, and $l_0 = 50\ cm$ is its length.

3. ***Dependence T(l).***

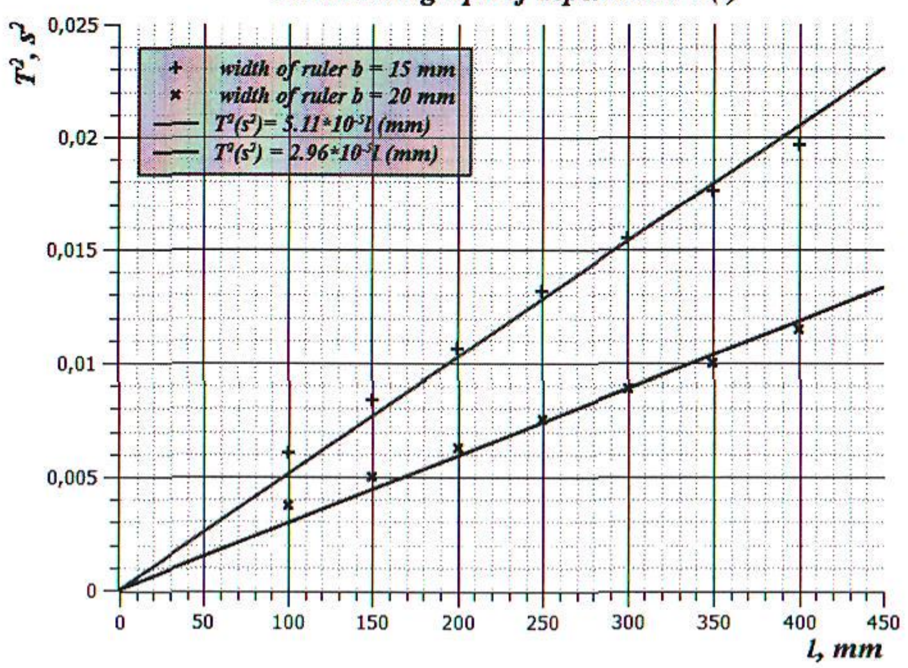
It is natural to assume that the modulus of torsion, by analogy with a coefficient of stiffness of usual spring, is inversely proportional to the length of ruler l $(q = -1)$:

$$k \sim \frac{1}{l} \rightarrow T \sim (l)^{1/2}, \text{ or } T^2 \sim l.$$

4. ***Comparison of experiment with theory.***

In coordinates $(l,\ T^2)$ a graph of dependence $T(l)$, which is assumed in point 3, has a form of straight line. For an experimental confirmation of our hypothesis we will construct the graph of $T(l)$ using the data of Tables 1 and 2, and putting the values of T^2 on the OY axis and the length of fixed part of ruler ℓ on the OX axis.

Linearised graph of dependence T(l)



It is seen that the experimental results are well described by a linear dependence of $T^2 = Bl$ with the proportionality coefficient $B_1 = 2.96\ s^2/mm$ for the clamped ruler with the width $b_1 = 20\ mm$, and $B_2 = 5.11\ s^2/mm$ for the ruler with the width $b_2 = 15\ mm$.
Hence, the experiment confirms our assumption:
$q = -1$.

5. ***Dependence of stiffness k on width of ruler b (p = ?)***

A dependence of stiffness on the width of ruler can be obtained by comparing the coefficients of proportionality B in the linearized dependencies $T^2 = Bl$ for the rulers of different widths (see point 4).

Indeed, if one assumes that the moment of inertia of pendulum is determined only by the moment of inertia of transverse ruler of width b, then:

$$I = \frac{Ml_0^2}{12} = \frac{\rho\delta b l_0^3}{12},$$

where ρ is the density of aluminium.
Since the lengths l_0 and thicknesses δ of rulers are the same, then the period of oscillations in the case of clamped ruler b_1:

$$T^2 = \frac{4\pi^2 I_2}{k_1} = \frac{4\pi^2 \rho\delta b_2 l_0^3}{12\beta G^m \delta^n b_1^p} l = B_1 l,$$

$$\left(I_2 = \frac{M_2 l_0^2}{12} = \frac{\rho\delta b_2 l_0^3}{12}\right),$$

and for the clamped ruler b_2:

$$T^2 = \frac{4\pi^2 I_1}{k_2} = \frac{4\pi^2 \rho\delta b_1 l_0^3}{12\beta G^m \delta^n b_2^p} l = B_2 l, \quad \left(I_1 = \frac{M_1 l_0^2}{12} = \frac{\rho\delta b_1 l_0^3}{12}\right).$$

A ratio of coefficients of proportionality:

$$\frac{B_2}{B_1} = \left(\frac{b_1}{b_2}\right)\left(\frac{b_1}{b_2}\right)^p$$

or, using the results of measurements:

$$\left(\frac{20}{15}\right)^p = \left(\frac{5.11}{2.96}\right)\left(\frac{15}{20}\right) \to (1.333)^p = 1.295 \to p$$

≈ 0.9.

Taking into account that p is an integral number, we obtain:

$$p = 1$$

7. **$m = ?$, $n = ?$**

We find exponents m and n by the method of dimensions taking into account that the dimension of shear modulus, as it follows from its definition, $[G] = Pa$.

The method of dimensions gives:

$m = 1$, $n = 3$.

Finally, a formula for the modulus of torsion of ruler looks like this:

$$k = \frac{G\delta^3 b}{3l} \qquad (5)$$

(A strict theory confirms this result (see Theory of elasticity, L. D. Landau, E. M. Lifshits))

8. ***Experiment for determining modulus of torsion of ruler of width of b = 20 mm.***

An experiment can be put this way:

1. we will determine the length of clamped ruler of width $b = 20$ *mm*, for example, $l = 300$ *mm*. We will measure the frequency (period) of torsional oscillations ν_0 (the period $T_0 = 1/\nu_0$).

2. we will attach to ends of oscillating (transverse) ruler a pair of magnets on each edge. Due to an increase of moment of inertia the oscillation frequency ν will decrease, and the period, respectively, will increase and will become equal to $T > T_0$.

3. From a formula for the period of torsional oscillations, it follows that:

$$T^2 - T_0^2 = \frac{4\pi^2 \Delta I}{k},$$

where $\Delta I = ml_0^2$ is a change of moment of inertia of system with the magnets ($m = 3.75$ *g* is the mass of one magnet) → the modulus of torsion is calculated according to a formula:

$$k = \frac{4\pi^2 \Delta I}{T^2 - T_0^2} = \frac{4\pi^2 m l_0^2}{T^2 - T_0^2} \qquad (6)$$

Results of measurements:

1. $\nu_0 = 642.2\ \text{min}^{-1} \to T_0 = 0.0934\ s \to T_0^2 = 8.73\cdot10^{-3}\ s^2$
2. $\nu = 443\ \text{min}^{-1} \to T = 0.135\ s \to T^2 = 18.3\cdot10^{-3}\ s^2$
3. $k = \frac{4\pi^2 m l_0^2}{T^2 - T_0^2} = \frac{4\cdot3.14^2\cdot4\cdot10^{-3}\cdot0.5^2}{(18.3-8.73)10^{-3}} = 4.12 J/rad.$

The modulus of torsion of ruler of width $b = 20$ *mm*:

k = 4.12 *J/rad.*

6. **$G = ?$**

The shear modulus of material of ruler (aluminium) we will calculate according to a formula:

$$G = \frac{3lk}{\delta^3 b}. \qquad (7)$$

We substitute results of measurements:

k = 4.12 J/rad; l= 300 mm; δ = 2 mm; b = 20 mm →

$$G = \frac{3lk}{\delta^3 b} = \frac{3\cdot0.3\cdot4.12}{2^3\cdot10^{-9}\cdot2\cdot10^{-2}} \approx 2.3\cdot10^{10}\,Pa = 3\cdot0.3\cdot4.12/2^3\cdot10^{-9}\cdot2*10^{-2} \approx 2.3*10^{10}\,Pa.$$

(for reference: $G_{table} = (2.5 - 2.6)\cdot10^{10}\ Pa$)

Grading system

Tables 1 and 2 in the presence of at least 4 points **3**

Tables 1 and 2 in the presence of not less than 7 points **6**

From the equation of harmonic oscillations the formula for the oscillation period is obtained and the expression for the moment of inertia of pendulum is written..**2**

The conclusion is made that the modulus of torsion is inversely proportional to the length of ruler l..**2**

The linear dependences $T(l)$ are made according to the experimental data **2**

The theoretical dependences $T(l)$ are made and the coefficients B1 and B2 are determined with precision up to 20% **2**

The theoretical dependences $T(l)$ are made and the coefficients B1 and B2 with precision up to 10% **3**

The exponent p is determined with precision up to 20%...........**3**

The exponent p is determined with precision up to 10%...........**5**

Formula (5) is obtained for the modulus of torsion of ruler **3**

The experiment is properly conducted, which allows determining the numerical value of modulus of torsion κ for one of rulers and formula (6) is obtained...**4**

The shear modulus of material of ruler is calculated by formula (7) with precision exceeding 20%..**1**

The shear modulus of material of ruler is calculated by formula (7) with precision up to 20%..**3**

"Tuymaada - 2016"
Efimov V.V.

Problem of experimental round № 2.

Calculate a density of crystal of table salt (the salt must not be soaked).
Equipment: syringes of 10 ml and 20 ml with caps for holes, plasticine, a plastic tube of length of 1 m (from a dropping bottle), a vessel with colored water, a container with salt, a ruler, Scotch tape, a cork for a tube.

Possible variant of solution.
1. To close holes of syringes by the caps, to remove pistons from them. Using plasticine as a stand, to make from the syringe a measuring cylinder of 20 ml, to pour into it ≈ 10 ml of water. To immerse the syringe up to 10 ml at first empty and then with salt. After measuring a volume of water displaced by salt and a volume of salt, to calculate an average density of salt ρ_{cp}.

To put a drop of water at the beginning of tube. To connect to the tube the syringe for 20 ml with a volume of air of 20 ml under the piston, moving the piston as far as it will go, to measure a shift of drop and to calculate a volume of tube of length of 1 m.

To put again a drop at the beginning of tube, to plug the end of tube by the cork. To pour into the syringe 10 ml of salt, to place the piston at a point of 20 ml, to connect the syringe to the beginning of tube. To move the piston to contact with salt, to measure the shift of drop of water. Using the Boyle–Mariotte law for air in the tube and air in the syringe, one can calculate the volume of salt without air.

2. For air in the syringe

$$(20 - V_c)P_0 = (10 - V_c + \Delta V)P \qquad (1)$$

3. For air in the tube

$$V_{mp}P_{в} = (V_o - \Delta V)P \qquad (2)$$

Dividing one equation by the other equation, we will obtain

4. $$\frac{20 - V_c}{V_{mp}} = \frac{10 - V_c + \Delta V}{V_{mp} - \Delta V} \qquad (3)$$

5. $$\rho_{кр.c} = \frac{m_c}{V_c} = \frac{10\rho_{cp}}{V_c} \qquad (4)$$

Grading system

The average density of salt is found 2
Formula (1) for air in the syringe is obtained 3
Formula (2) for air in the tube is determined.............................3
Formula (3) is obtained..2
Formula (4) for the density of salt is found 2
An error estimate..3

"Tuymaada - 2016"
Efimov V.V.

Problem of experimental round № 1.

Compute with maximum accuracy a length of insulating tape in a roll and its thickness without unwinding the roll.
Equipment: the roll of insulating tape, a ruler with a length of 20-30 cm with a strip of insulating tape, which is glued on along the whole length of ruler.

Possible variant of solution:

1. To wind the insulating tape from the ruler in a cylinder.
2. To roll the cylinder on the ruler, counting a number of revolutions N
3. To calculate a radius of cylinder and an area of its base

$$r = \frac{l}{2\pi N}; \; S_ц = \pi r^2 = \frac{l^2}{4\pi N^2}$$

4. To measure with the ruler an inner and outer diameters of roll of insulating tape and to calculate a sectional area of roll

$$S_p = \frac{\pi(D^2 - d^2)}{4}$$

5. To calculate a length of insulating tape

$$L = \frac{lS_p}{S_ц} = \frac{\pi(D^2 - d^2)4\pi N^2}{4l^2} = \frac{\pi^2(D^2 - d^2)N^2}{l}$$

6. To calculate a thickness of insulating tape

$$h = \frac{S_ц}{l} = \frac{l}{4\pi N^2}$$

Grading system

The number of revolutions N is counted	3
The radius of cylinder and the area of its base are determined	3
The sectional area of roll is calculated	2
The length of insulating tape is determined	2
The thickness of insulating tape is found	2
An error estimate	3