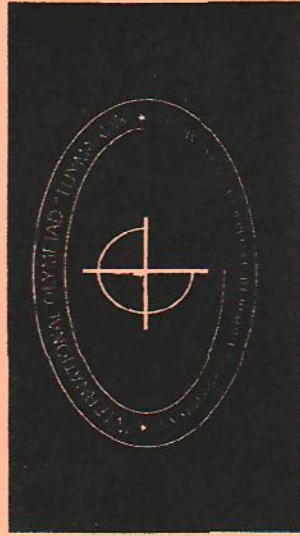


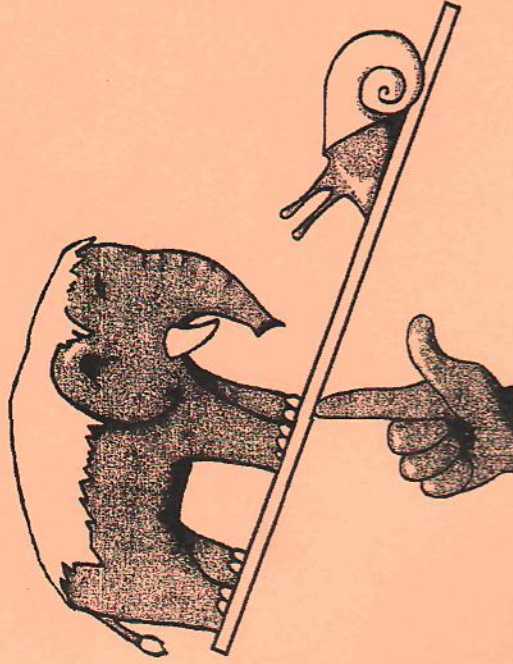
Министерство образования Республики Саха (Якутия)
Физико-математический форум "Ленский край"
Якутский государственный университет им. М.К.Аммосова

XII Международная физическая олимпиада "Туймаада"

XII International physics olympiad "Tuymaada"



Физика



Якутск, 7-15 июля 2005 г.

Problem 1 (A discrete model of avalanche formation)

A heavy bricks are situated on the long inclined plane with slope angle α . The bricks are separated from each other by distance L (Fig. 1). They are held in the fixed state by the rest frictional force. Let us suppose that the brick starts motion when it acquires an infinitesimal impulse and that its further motion is frictionless. If the highest brick starts motion it will collide with the second brick, after a chain of two bricks will collide with the third brick and so on. All collisions are nonelastic. So a long chain of bricks arises to which new bricks join continuously. This process simulates an avalanche motion at the mountainside.

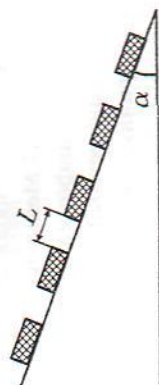


Fig. 1

- Let the moving chain has $n = 100$ bricks. Determine the chain velocity increase Δv_1 right after the impact with 101 brick compared to the velocity right after the impact with 100 brick.
- Determine the difference in velocities Δv_2 of chains containing 100 and 400 bricks.
- What will change in the avalanche motion if the frictional forces are taken into account? Assuming that the avalanche starts when the mountain slope $\alpha \geq 23^\circ$, give answers to the previous two items taking into account the frictional force. In this item you could use the following value of the mountain slope $\alpha = 30^\circ$.

Problem 2 (Mars opposition)

In the following problem you should investigate two planets of the Solar system motion: Earth and Mars. Opposition is the moment when two planets are situated on the same line originating from the Sun.

- Assuming that the planets moves on concentric circles around the Sun determine the time interval τ between two subsequent oppositions of Earth and Mars. The ratio of Earth's and Mar's orbits equals: $R_M/R_E = 1.52$. Earth's year duration equals $T_E = 365$ days.
- In Fig. 2 an opposition of Mars and Earth is depicted. Show at the same figure the position of the planets in the next opposition. The planets move clockwise.

Actually the time interval between the two subsequent oppositions lies in the certain interval due to the ellipticity of Mars orbit. To each moment when the

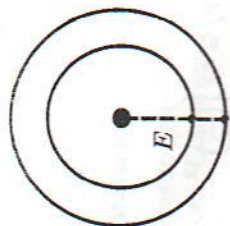


Fig. 2

opposition occurs corresponds a determined time interval before the next opposition, as follows from the constant orientation of Earth's and Mars orbits in the space. This dependency is shown in Figure 3. In the further considerations you may assume that orbit of Earth is circular and orbit of Mars possesses a small ellipticity. The following data is given: the Earth's orbit radius equals $R_E = 1.50 \times 10^{11}$ m, a big half-axe of the Mars's orbit equals $R_M = 1.52 R_E$, the Earth's year duration equals $T_E = 365$ days.

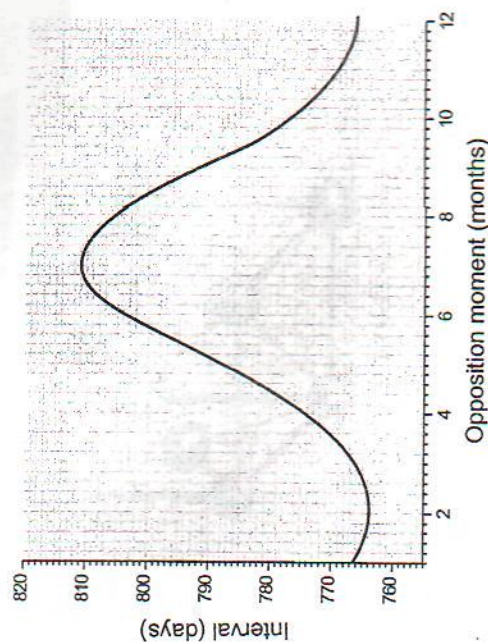


Fig. 3

- Determine the maximum R_{max} and the minimum R_{min} distances between Mars and the Sun.

Problem 3 (Molecular crystals)

Only properties of the simplest molecular crystals formed by the atoms of inertia gases will be considered in this problem. Consideration will be carried on the example of argon atom Ar. The molecular crystals atoms are consolidated by very weak Van der Waals forces. Two atoms interaction in the crystal is described by the Lennard-Jones potential:

$$U(r) = 4\varepsilon \left[\left(\frac{\sigma}{r} \right)^{12} - \left(\frac{\sigma}{r} \right)^6 \right],$$

where r is the distance between the atoms. For argon atoms $\varepsilon = 0.00104$ eV, $\sigma = 3.40$ Å.

- Plot schematically the dependence $U(r)$.
- Determine equilibrium separation between two isolated atoms R_0 as function of σ if interaction between them is described by Lennard-Jones potential. Give a numerical value of the separation.

During the following consideration we will treat solid argon as an assembly of classical particles localized in the points of the face-centered cubic lattice (Fig. 4). The kinetic energy of the particles is negligibly small. Let's look at an atom of our argon crystalline structure.

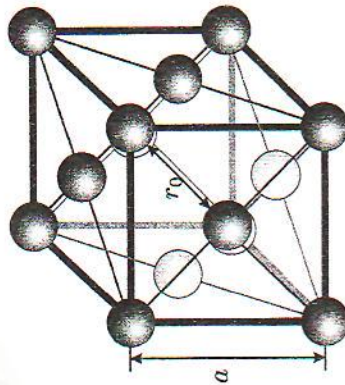


Fig. 4

- The interaction potential energy of the singled out atom with the rest of the crystal is given by the formula:

$$U = 4\varepsilon \left[A \left(\frac{\sigma}{r_0} \right)^{12} - B \left(\frac{\sigma}{r_0} \right)^6 \right],$$

where r_0 is the distance between the nearest neighbors. Obtain the numerical values for the coefficients A and B taking into account contribution only from six groups of atoms (each group contains atoms equally spaced from the singled out atom).

- Determine the equilibrium separation between the nearest neighbors r_1 from the previous item as function $r_1(\sigma, A, B)$.
- Determine the lattice distance a (Fig. 4) for argon crystals.

Measurements of the crystal elastic properties is a common knowledge tool for the internal structure investigations. Compression modulus κ is a physical quantity defined as follows:

$$\kappa = -V \frac{dp}{dV}.$$

If the crystal has N atoms then its potential energy u has the following form:

$$u = 2N\varepsilon \left[A \left(\frac{\sigma}{r_1} \right)^{12} - B \left(\frac{\sigma}{r_1} \right)^6 \right].$$

The origin of the factor 2 here is connected to the twice considered interaction energy of each atoms pair.

- Determine the compression modulus κ for argon crystals as a function $\kappa(\varepsilon, \sigma, A, B)$.
- Obtain the numerical value of κ for argon crystals.

Problem 4 (Oscillations in the circuit with diodes)

The circuit in Fig. 5 contains two capacitors of capacitance C_1 and C_2 , two inductance coils of inductance L_1 and L_2 , two ideal diodes D_1 and D_2 , and a switch K . Initially capacitor C_2 is charged to the voltage U_0 . In the moment $t = 0$ the switch K is locked.

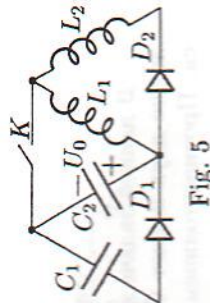


Fig. 5

- Determine duration τ of the transient process (that is the moment τ , from which the process starts to be periodic).
- Determine the oscillation period T in the stationary regime.
- Find the voltages U_1 and U_2 on the capacitor C_2 in the moments when for the first and second time there is no current through it.
- Determine the oscillatory voltage amplitude A on the capacitor C_2 in the stationary regime.
- Sum up results of the previous items by plotting schematically the dependence of voltage U on the capacitor C_2 versus time t in the range from 0 to $\tau + T$. Denote key points (such as maximums, minimums, and crossing points).

High league

Problem 1 (A discrete model of avalanche formation)

Let the chain velocity right after the collision with n brick be v_n . The chain acceleration may be written as $a = g \sin \alpha$. Using the impulse conservation law for the collision process and the energy conservation law for the motion between two collisions we find:

$$v_{n+1}^2 = \left(\frac{n}{n+1} \right)^2 (v_n^2 + 2gL \sin \alpha) \quad (1.0 \text{ point}).$$

Analogously:

$$v_n^2 = \left(\frac{n-1}{n} \right)^2 (v_{n-1}^2 + 2gL \sin \alpha).$$

Recursive substituting yields:

$$v_{n+1}^2 = \frac{2gL \sin \alpha}{(n+1)^2} (n^2 + (n-1)^2 + \dots + 1^2) \quad (2.0 \text{ points}).$$

It is worth mentioning that during solution we deal with $n \gg 1$, thus the sum in brackets can be replaced by the integral:

$$v_{n+1}^2 \approx \frac{n^3}{3(n+1)^2} 2gL \sin \alpha \approx \frac{n^3}{3n^2} 2gL \sin \alpha \approx 2 \frac{g \sin \alpha}{3} nL \quad (3.0 \text{ points}).$$

Now it is clear that the chain moves with an average acceleration $a^* = g \sin \alpha / 3$. Consequently:

$$\Delta v_1 = \sqrt{\frac{2g(n+1)L \sin \alpha}{3}} - \sqrt{\frac{2gnL \sin \alpha}{3}} \approx \sqrt{\frac{gL \sin \alpha}{6n}} \quad (1.0 \text{ point}).$$

The difference in velocities Δv_2 of chains containing 100 and 400 bricks can be obtained as follows:

$$\Delta v_2 = \sqrt{\frac{800}{3}} gL \sin \alpha - \sqrt{\frac{200}{3}} gL \sin \alpha \quad (1.0 \text{ point}).$$

Due to $30^\circ > 23^\circ$ the avalanche starts motion. From the given data one can readily obtain the expression for the frictional coefficient $\mu = \tan \beta$, where $\beta = 23^\circ$. To answer the previous two items questions one have to replace g by $g(1 - \tan \beta \cos \alpha)$ (2.0 points).

Problem 2 (Mars opposition)

Due to the Kepler's law Mars period of revolution T_M is:

$$\frac{T_M^2}{T_E^2} = \frac{R_M^3}{R_E^3}, \quad \text{hence} \quad T_M = T_E \left(\frac{R_M}{R_E} \right)^{3/2} \quad (1.0 \text{ point}).$$

The next opposition will take place when Earth produces an entire revolution in Mars framework:

$$\left(\frac{2\pi}{T_E} - \frac{2\pi}{T_M} \right) \tau = 2\pi, \quad \text{hence} \quad \tau = T_E \frac{R_M^{3/2}}{R_M^{3/2} - R_E^{3/2}} \quad (1.0 \text{ point}).$$

Substituting a numerical data:

$$\tau \approx 783 \text{ days} \quad (0.5 \text{ points}).$$

Let φ be an angular displacement of the planets relative to the Sun in the next opposition. Then taking into account that Mars produces only one revolution during τ :

$$\varphi = 2\pi \left(\frac{\tau}{T_M} - 1 \right), \quad \text{hence} \quad \varphi = 2\pi \left(\frac{\tau}{T_E} \left(\frac{R_E}{R_M} \right)^{3/2} - 1 \right) \quad (1.0 \text{ point}).$$

Substituting a numerical data:

$$\varphi \approx 52^\circ \quad (0.5 \text{ points}).$$

Let the last opposition happened virtually in the perihelion when the planets are situated in E_1 and M_1 (Fig. 17), and the next opposition when the planets are situated in E_2 and M_2 . Let section $E_1 - E_2$ passing time be t_1 , and section

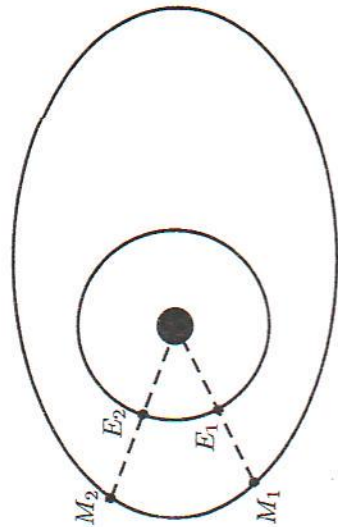


Fig. 13

High league. Possible solutions

16

$M_1 - M_2$ passing time be t_2 . Then the following equation is valid:

$$2T_E + t_1 = T_M + t_2, \text{ where } T_M = T_E \left(\frac{R_M}{R_E} \right)^{3/2} = 684 \text{ days} \quad (1.0 \text{ point}).$$

The Earth v_E and Mars v_M velocities on the sections $E_1 - E_2$ and $M_1 - M_2$ are proportional:

$$v_E \sim \frac{1}{\sqrt{R_E}}, \quad v_M \sim \frac{1}{\sqrt{R_{min}}} \quad (2.0 \text{ points}).$$

Due to the relation between the lengths of the sections:

$$\frac{l_{E_1 E_2}}{l_{M_1 M_2}} = \frac{R_E}{R_{min}}, \text{ hence } \frac{t_1}{t_2} = \left(\frac{R_E}{R_{min}} \right)^{3/2} \quad (1.0 \text{ point}).$$

Since time interval between two subsequent opposition equals $\tau_{max} = 2T_E + t_1$:

$$2T_E + t_1 = T_M + t_1 \left(\frac{R_{min}}{R_E} \right)^{3/2}, \text{ hence } t_1 = \frac{2T_E - T_M}{\left(\frac{R_{min}}{R_E} \right)^{3/2} - 1},$$

$$\tau_{max} = 2T_E + \frac{2T_E - T_M}{\left(\frac{R_{min}}{R_E} \right)^{3/2} - 1}, \text{ hence } R_{min} = R_E \left(\frac{\tau_{max} - T_M}{\tau_{max} - 2T_E} \right)^{2/3} \quad (1.0 \text{ point}).$$

The similar equations may be written for R_{max} . Substituting a numerical data:

$$\begin{aligned} \tau_{max} &= 811 \text{ days} & R_{min} &= 1.35 R_E = 2.03 \times 10^{11} \text{ m}, & (0.5 \text{ points}) \\ \tau_{min} &= 764 \text{ days} & R_{max} &= 1.77 R_E = 2.66 \times 10^{11} \text{ m} & (0.5 \text{ points}). \end{aligned}$$

Problem 3 (Molecular crystals)

Sketchy plot of $U(r)$ is shown in Fig. 14 (1.0 point). It is obvious that on large distances a small attraction force is acting between two atoms and on small distances a huge repulsion force. To find an equilibrium distance between two atoms one must equate the derivative of the potential energy to nought:

$$\left. \frac{dU}{dr} \right|_{r=R_0} = 0, \text{ hence } R_0 = \sqrt[9]{2}\sigma \quad (1.0 \text{ point}).$$

For argon atoms substituting $\sigma = 3.40 \text{ \AA}$ we obtain that $R_0 = 3.82 \text{ \AA}$ (0.5 points).

In fact our potential energy U equals:

$$U = \sum_i 4\epsilon \left[\left(\frac{\sigma}{r_i} \right)^{12} - \left(\frac{\sigma}{r_i} \right)^6 \right],$$

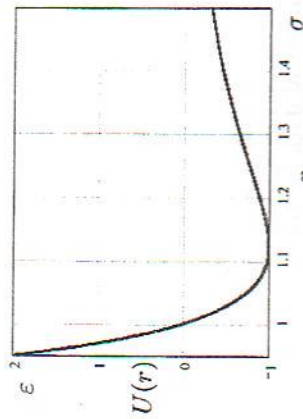


Fig. 14

where r_i is the separation between singled out atoms and the rest atoms of the crystalline lattice. Each singled out atom has the following neighbors:

Neighbor number	12	6	24	12	12	16
Neighbor distance	r_0	$\sqrt{2}r_0$	$\sqrt{3}r_0$	$2r_0$	$\sqrt{5}r_0$	$\sqrt{7}r_0$

Then it's easy to find that:

$$\begin{aligned} A &= 12 + 6 \frac{1}{(\sqrt{2})^{12}} + 24 \frac{1}{(\sqrt{3})^{12}} = 12 + 0.096 + 0.033 = 12.1 \quad (1.5 \text{ points}), \\ B &= 12 + 6 \frac{1}{(\sqrt{2})^6} + 24 \frac{1}{(\sqrt{3})^6} + 12 \frac{1}{2^6} + 12 \frac{1}{(\sqrt{5})^6} = \\ &= 12 + 0.75 + 0.888 + 0.192 + 0.096 + 0.047 = 13.9 \quad (1.5 \text{ points}). \end{aligned}$$

To find an equilibrium distance between the nearest neighbors r_1 one must equate to nought the differential:

$$\left. \frac{dU}{dr} \right|_{r=r_1} = 0, \text{ hence } 12A \frac{\sigma^{12}}{r_1^{13}} = 6B \frac{\sigma^6}{r_1^7}, \quad r_1 = \sqrt[6]{\frac{2A}{B}} \sigma \quad (1.0 \text{ point}).$$

The lattice constant equals $a = \sqrt{2}r_1$, substituting the numerical data for the argon atoms $a = 3.73 \text{ \AA}$ (0.5 points).

Due to the conservation law of energy (the kinetic energy is negligible) $p dV = -du$. Therefore the above equation yields:

$$p = -V \frac{dp}{dV} = V \frac{d^2 u}{dV^2} \bigg|_{r_0=r_1} \quad (1.0 \text{ point}).$$

The elementary cell a^3 consists of 4 atoms (Fig. 4), therefore the net crystal volume equals (0.5 points):

$$V = \frac{N}{4} a^3 = \frac{N\tau_0^3}{\sqrt{2}} \Rightarrow u(V) = 2N\varepsilon \left(A \frac{N^4 \sigma^{12}}{4V^4} - B \frac{N^2 \sigma^6}{2V^2} \right),$$

$$\frac{d^2 u}{dV^2} = 2N\varepsilon \left(5A \frac{N^4 \sigma^{12}}{V^6} - 3B \frac{N^2 \sigma^6}{V^4} \right) \Rightarrow \frac{d^2 u}{dV^2} \Big|_{r_0=r_1} = 2N\varepsilon \left(40A \frac{\sigma^{12}}{N^2 r_1^{18}} - 12B \frac{\sigma^6}{N^2 r_1^{12}} \right)$$

$$r_1 = \sqrt[6]{\frac{A}{2B} \sigma} \Rightarrow \alpha = 2\varepsilon \frac{\tau_1^3}{\sqrt{2}} \frac{2B^3}{A^2 \sigma^6} = \frac{4\varepsilon}{\sigma^3} \frac{B^{5/2}}{A^{3/2}} \quad (1.0 \text{ point}).$$

Substituting the numerical data for the argon atoms:

$$\alpha = 2.89 \times 10^9 \frac{\text{N}}{\text{m}^2} \quad (0.5 \text{ points}).$$

Problem 4 (Oscillations in the circuit with diodes)

After the switch K is locked (Fig. 15) capacitor C_2 starts to discharge through the parallel inductance coils L_1 and L_2 . Diode D_2 doesn't hinder this process, and diode D_1 doesn't allow capacitor C_1 to be charged. The charge of the capacitor C_2 reaches zero in the moment $\tau_1 = T_1/4$, where T_1 is the oscillation period of the circuit comprised of capacitor C_2 and coil $L = L_1 L_2 / (L_1 + L_2)$:

$$T_1 = 2\pi \sqrt{\frac{L_1 L_2}{L_1 + L_2} C_2}, \quad \text{hence} \quad \tau_1 = \frac{\pi}{2} \sqrt{\frac{L_1 L_2}{L_1 + L_2} C_2} \quad (1.0 \text{ point}).$$

From the moment τ_1 the coils start to charge parallel capacitors C_1 and C_2 , since diode D_1 doesn't resist current flow in this direction. The duration of charging equals $\tau_2 = T_2/4$, where T_2 is the oscillation period of the circuit comprised of capacitor $C = C_1 + C_2$ and inductance coil L :

$$T_2 = 2\pi \sqrt{\frac{L_1 L_2}{L_1 + L_2} (C_1 + C_2)}, \quad \text{hence} \quad \tau_2 = \frac{\pi}{2} \sqrt{\frac{L_1 L_2}{L_1 + L_2} (C_1 + C_2)} \quad (1.0 \text{ point}).$$

Due to the energy conservation law applied to the initial and final moments:

$$\frac{C_2 U_0^2}{2} = \frac{(C_1 + C_2) U_1^2}{2}, \quad \text{hence} \quad U_1 = U_0 \sqrt{\frac{C_2}{C_1 + C_2}} \quad (1.0 \text{ point}).$$

For any further process current through the diode D_1 equals zero because the voltage on capacitor C_2 can not exceed U_1 , consequently the charge of C_1 is a constant.

Then, capacitor C_2 will discharge through coil L_1 (as far as diode D_2 switches off coil L_2) to the zero voltage during the time interval $\tau_3 = T_3/4$, where T_3 is the oscillation period of the circuit comprised of capacitor C_2 and inductance coil L_1 :

$$T_3 = 2\pi \sqrt{L_1 C_2}, \quad \text{hence} \quad \tau_3 = \frac{\pi}{2} \sqrt{L_1 C_2} \quad (1.0 \text{ point}).$$

When capacitor C_2 discharges, the current through L_1 will reach the value:

$$I_1 = U_1 \sqrt{\frac{C_2}{L_1}}.$$

Then the coil L_1 energy distributes between capacitor C_2 and coil L_2 . When the current through the capacitor becomes zero, its voltage will reach the maximum value U_2 . At the same time current I will run in the circuit containing two coils and diode D_2 . The derivatives of fluxes through the coils are equal since from the moment $\tau_1 + \tau_3 + \tau_3$ EMF of self-induction on coils L_1 and L_2 are identical:

$$d\Phi_1 = d\Phi_2, \quad \text{or} \quad L_1(I - I_1) = -L_2 I, \quad \text{hence} \quad I = \frac{L_1}{L_1 + L_2} I_1 \quad (1.0 \text{ point}).$$

From the energy conservation law written for this stage of the process:

$$\frac{C_2 U_1^2}{2} = \frac{C_2 U_2^2}{2} + \frac{(L_1 + L_2) I^2}{2},$$

$$U_2 = U_1 \sqrt{\frac{L_2}{L_1 + L_2}} = U_0 \sqrt{\frac{C_2}{C_1 + C_2}} \sqrt{\frac{L_2}{L_1 + L_2}} \quad (1.0 \text{ point}).$$

Further oscillations in the circuit comprised of capacitor C_2 and coils connected in parallel will be superimposed on the constant current I running through two coils and diode D_2 connected in series, thus D_2 will operate in a continuous regime. Therefore, from the moment $\tau = \tau_1 + \tau_3 + \tau_3$ (1.0 point), the process becomes periodic with periodicity (Fig. 16) (1.0 point):

$$T = 2\pi \sqrt{\frac{L_1 L_2}{L_1 + L_2} C_2} \quad (1.0 \text{ point})$$

and voltage amplitude:

$$A = U_2 = U_0 \sqrt{\frac{C_2}{C_1 + C_2}} \sqrt{\frac{L_2}{L_1 + L_2}} \quad (1.0 \text{ point}).$$

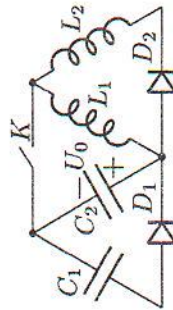


Fig. 15

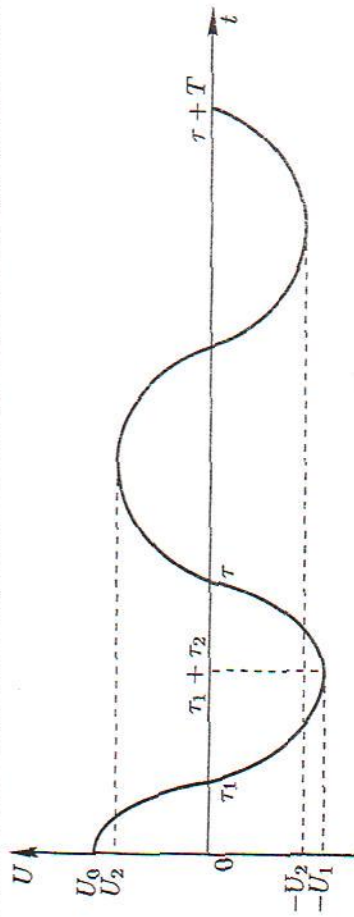


Fig. 16

Высшая лига

Задача 1 (Дискретная модель движения лавины)

Обозначим скорость цепочки сразу после соударения с n -ым кубиком v_n . Ускорение, с которым движется цепочка между столкновениями равно $a = g \sin \alpha$. Используя закон сохранения импульса для процесса столкновения и закон сохранения энергии для движения между столкновениями находим:

$$v_{n+1}^2 = \left(\frac{n}{n+1} \right)^2 (v_n^2 + 2gL \sin \alpha) \quad (1.0 \text{ балл}).$$

Аналогично можно записать:

$$v_n^2 = \left(\frac{n-1}{n} \right)^2 (v_{n-1}^2 + 2gL \sin \alpha).$$

Рекуррентно подставляя находим:

$$v_{n+1}^2 = \frac{2gL \sin \alpha}{(n+1)^2} (n^2 + (n-1)^2 + \dots + 1^2) \quad (2.0 \text{ балла}).$$

Поскольку при ответе на пункты задания $n \gg 1$, сумму в скобках можно заменить интегралом и тогда:

$$v_{n+1}^2 \approx \frac{n^3}{3(n+1)^2} 2gL \sin \alpha \approx \frac{n^3}{3n^2} 2gL \sin \alpha \approx 2 \frac{gL \sin \alpha}{3} nL \quad (3.0 \text{ балла}).$$

Видно, что цепочка движется со средним ускорением $a^* = g \sin \alpha / 3$. Таким образом:

$$\Delta v_1 = \sqrt{\frac{2g(n+1)L \sin \alpha}{3}} - \sqrt{\frac{2gnL \sin \alpha}{3}} \approx \sqrt{\frac{gL \sin \alpha}{6n}} \quad (1.0 \text{ балл}).$$

Для разности скоростей цепочек из 100 и 400 кубиков имеем:

$$\Delta v_2 = \sqrt{\frac{800}{3} gL \sin \alpha} - \sqrt{\frac{200}{3} gL \sin \alpha} \quad (1.0 \text{ балл}).$$

Поскольку $30^\circ > 23^\circ$, то лавина придет в движение. Из условия следует, что коэффициент трения скольжения лавины по склону равен $\mu = \tan \beta$, где $\beta = 23^\circ$. Тогда для ответа на два предыдущих пункта необходимо везде в формулах заменить g на $g(1 - \tan \beta \cdot \cos \alpha)$ (2.0 балла).

Задача 2 (Противостояние Марса)

По законам Кеплера легко найти период обращения Марса по своей орбите T_M :

$$\frac{T_M^2}{T_E^2} = \frac{R_M^3}{R_E^3}, \quad \text{откуда} \quad T_M = T_E \left(\frac{R_M}{R_E} \right)^{3/2} \quad (1.0 \text{ балл}).$$

Следующее противостояние наступит, когда Земля совершит полный оборот в системе отсчета Марса:

$$\left(\frac{2\pi}{T_E} - \frac{2\pi}{T_M} \right) \tau = 2\pi, \quad \text{откуда} \quad \tau = T_E \frac{R_M^{3/2}}{R_M^{3/2} - R_E^{3/2}} \quad (1.0 \text{ балл}).$$

Подставляя численные данные находим:

$$\tau \approx 783 \text{ сут} \quad (0.5 \text{ балла}).$$

Обозначим через φ угловое смещение радиус вектора планет относительно Солнца при очередном противостоянии. Тогда учитывая, что Марс делает за время τ всего один полный оборот:

$$\varphi = 2\pi \left(\frac{\tau}{T_M} - 1 \right), \quad \text{откуда} \quad \varphi = 2\pi \left(\frac{\tau}{T_E} \left(\frac{R_E}{R_M} \right)^{3/2} - 1 \right) \quad (1.0 \text{ балл}).$$

Подставляя численные данные находим:

$$\varphi \approx 52^\circ \quad (0.5 \text{ балла}).$$

Пуская последнее противостояние произошло практически в перигелии, когда положение планет E_1 и M_1 (Рис. 17), а следующее при положениях E_2 и M_2 . Пусть время прохождения участка $E_1 - E_2$ траектории Земли равно t_1 , а участка $M_1 - M_2$ Марсом t_2 . Тогда верно равенство:

$$2T_E + t_1 = T_M + t_2, \quad \text{где} \quad T_M = T_E \left(\frac{R_M}{R_E} \right)^{3/2} = 684 \text{ сут} \quad (1.0 \text{ балл}).$$

Photoresistor properties investigation

Equipment

During the experiment the following equipment is available: photoresistor SF 3-1 mounted on the rack, variable resistor, multimeter, battery 4.5 V, bulb on the rack (6 V - 3 W), ruled paper, logarithmic paper, darkening box, glassy plate, leads.

Introduction

A semiconductor device based on the photoelectric signal transformation are widely used in the modern technology. Mentioned devices form a basis of optoelectronics - a new scientific branch, in which optical as well as electrical means are used for storing, processing and transfer of information. In the following work you should investigate a photoresistor physical properties. The photoresistor is a semiconductor device which conductivity changes by the incident light. In the work a membraneous photoresistor is used which main part is a semiconductor plane (Fig. 27).

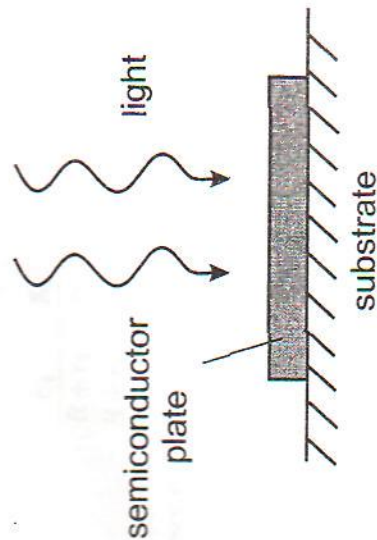


Fig. 27

Photoresistor electrical characteristics

- Measure volt-ampere characteristics of the given photoresistor under the room illumination.

Luminous (lux-ohmic) characteristics of the photoresistor

Apart from the measured electrical responses photoresistor properties are also described by the luminous characteristics.

- Reduce outer light radiation to the minimum possible value. After that obtain the luminous characteristics of the photoresistor: the dependency of resistance R on illumination E . (In your report the illumination axis must be plotted in the logarithmic scale)
- Using measured dependency find illumination in the room E_r .

The refractive index of the glassy plate

Fyrenel's formulas are usually used in the optics. One of them relates the intensity of the incident I_i and reflected I_r beams from the boundary of two mediums with refractive indexes n_1 and n_2 :

$$\frac{I_r}{I_i} = \left(\frac{n_1 - n_2}{n_1 + n_2} \right)^2$$

- Determine the refractive index n of the given glassy plate. A multiple reflection in the plate can be neglected

Photoresistor properties investigation

Photoresistor volt-ampere characteristics (Fig. 30) is measured by means of scheme shown in Fig. 29 under room illumination.

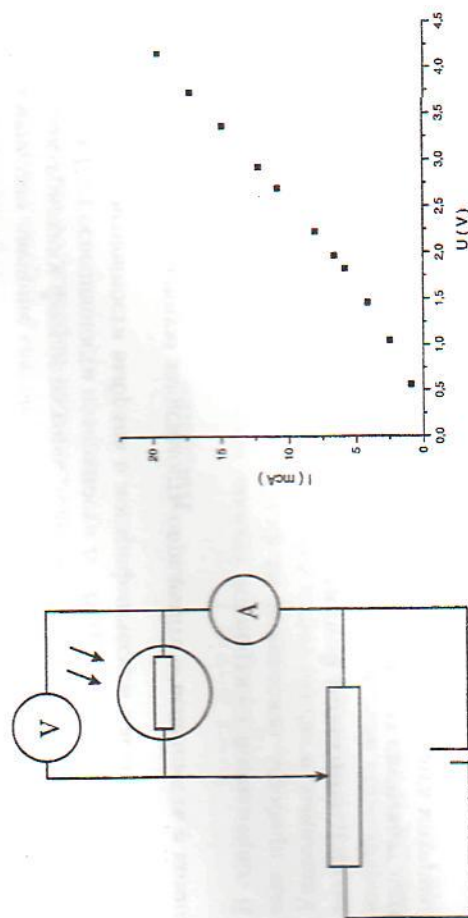


Fig. 29

Fig. 30

Connecting the bulb to the battery, and the photoresistor to the multimeter in the ohmic regime, we put an assembled construction into the darkening box. In further discussion the bulb is considered to be a point source with power $P = U_0 I_0$. In our experiments $U_0 = (3.70 \pm 0.01)$ V, $I_0 = (380 \pm 10)$ mA. Hence $P = (1.41 \pm 0.04)$ W. Let the distance between the photoresistor and the bulb be r , then the illumination E of the working surface is given by the formula:

$$E = \frac{P}{r^2}.$$

Therefore, varying distance between the bulb and the photoresistor lux-ohmic $R(E)$ characteristics can be obtained (Fig. 31).

Illumination E_r in the room may be obtained via lux-ohmic characteristics and photoresistor resistance R_r measured under the room radiation. In our experiments $R_r = (370 \pm 1)$ kOhm, thereby:

$$E_r = (77 \pm 2) \text{ lux}.$$

To measure the refractive index n the photoresistor is fixed at some distance from the bulb. In this case the photoresistor illumination E_1 is determined by means of the lux-ohmic curve and photoresistor resistance R_1 . After insertion

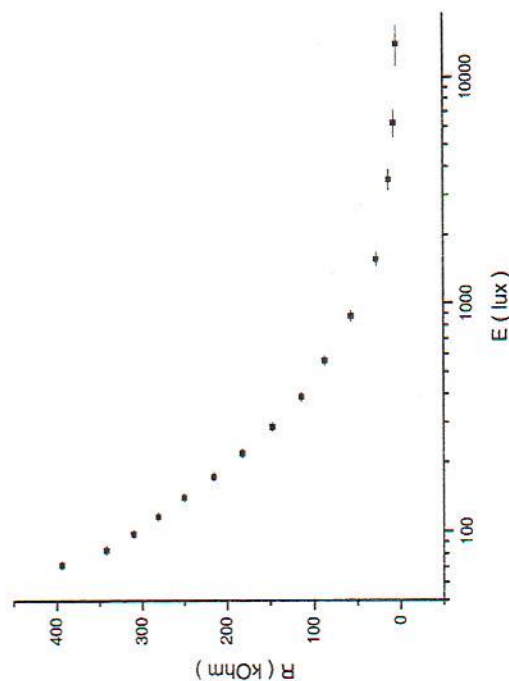


Fig. 31

of the glassy plate into the space between the photoresistor and the bulb the photoresistor illumination and resistance become respectively E_2 and R_2 . Due to Frenel's formula:

$$\frac{E_2}{E_1} = \left(1 - \left[\frac{n-1}{n+1} \right]^2 \right)^2,$$

which can be rewritten as:

$$n = \frac{1 + \left(1 - \sqrt{\frac{E_2}{E_1}} \right)^{1/2}}{1 - \left(1 - \sqrt{\frac{E_2}{E_1}} \right)^{1/2}}.$$

In our experiments $R_2 = (96.0 \pm 0.1)$ Ohm, $R_1 = (88.6 \pm 0.1)$ Ohm. Lux-ohmic curve yields $E_2 = (514 \pm 10)$ lux, $E_1 = (560 \pm 10)$ lux. Substituting into the formula for the refractive index:

$$n = (1.52 \pm 0.15).$$