Marking scheme

Senior league

Problem 1

Proved for some r that if $|XY| = r$ then $f(X) = f(Y)$: 2 points

Problem 3

Noted that if A and B defeated C, then they did not play against each other: 0 points

Problem 4

Proved that $a_1 + a_2 + \ldots + a_n = c$: 0 points

Noted that if RHS is changed at a point, then exactly one term of LHS is changed at this point: 0 points

The problem is reduced to equality of differences of two arithmetical progressions: 2 points

Problem 5

These criteria are NOT additive.

A. An example is given: 2 points

B. The bound is incorrect and/or incorrectly proved: not more than 2 points (points for the example included)

B1. The cases are not systematized. No explanation why all cases are done. One or several cases missing (when an example is given) – not more than 2 points (points for the example included)

B2. The "best" five is considered (when an example is given): not more than 2 points (points for the example included)

C1. The cases are systematized along the number of consecutive knights. The case of more than 3 liars after 3 knights is missing: not more than 4 points (points for the example included)

C2. The cases are systematized and are being checked taking into account symmetry in five consecutive persons: not more than 4 points (points for the example included)

D. The number of knights is estimated by looking on triplets, it is proved that at most 4 knights can be among 6 consecutive persons, but the final count is missing or incorrectly proved: 6 points

Problem 6

Proved that if x_1 is rational then the sequence contains 0: 3 points

Proved that if the number of non-zero terms is finite then their sum is an integer: 3 points

The fact that $x_n > x_{n+1}$ only for $n = \lceil \frac{1}{x_n} \rceil$ is used without proof: -1 балл

Problem 8

The cube presented is not filled entirely: 0 points

The fact that the constructed cube is Latin is not proved: 0 points

Младшая лига

Problem 1

Proved that if the number of multiples of a is m, then the number of multiples of $2a$ is at least $[m/2]$, or that if the number of multiples of b is m, then the number of multiples of $2b$ is at most $[(n + 1)/2]$, or both: 2 points

Problem 2

Proved that the circumcircles of (AQC) and (QBD) are equal: 0 points

The problem is reduced to the equality $AB \cdot BQ = QC \cdot CD$: 2 points

Proved that the chords on the extensions of BC are equal: 2 points

Problem 3

A correct example: 2 points

Proof of the bound: 5 points

Problem 5

For two numbers of different colours an arbitrarily long segment without the third colour is constructed: 2 points

Problem 6

The case of irrational x_1: 1 балл

Problem 7

Example: 3 points

Bound: 3 points