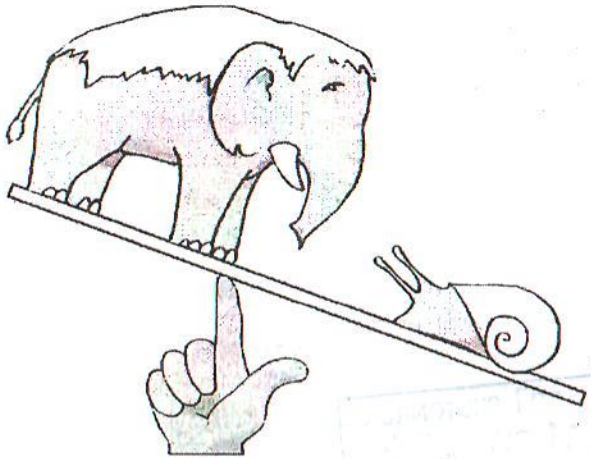
Министерство образования Республики Саха (Якутия)
Физико-математический форум «Ленский край»
Северо-восточный федеральный университет им. М.К. Аммосова

XXI Международная олимпиада «Туймаада»

Физика

Экспериментальный тур

Методическое пособие



Якутск, 5–11 июля 2014 г.

Junior league

Problem 1. Mechanical «grey box»

Determine the simplest scheme of the «grey box» and deflection rates k_1, k_2 and k_3 of each of springs it contains.

Equipment. «Grey box» with rods A and B, dynamometer, graph paper, clamps.

Note. One may stretch the springs applying force up to 5 N.

Problem 2. «Black box» with three terminals

1. Identify a possible scheme of combining elements inside the «black box», if it is known that it contains a nonlinear element, two resistors and a capacitor.
2. Determine the resistances of the resistors.
3. Determine the capacitance of the capacitor.

Equipment. The «black box», a battery, a multimeter, a variable resistor, a diode, a stopwatch, a resistor with the known resistance.

Note. The diode can be connected to the battery only through the resistor to avoid a short circuit.

Senior league

Problem 1. Eccentric fan

Introduction. When starting and stopping turbines, centrifuges, gyroscopes and other devices having a rapidly rotating rotor, at approaching close to some frequencies extreme vibrations appear and they stop after passing through a «forbidden zone» nearby a critical frequency. Observable effects of such vibrations are both a washing machine with a working centrifuge, innocently jumping all over the bathroom, and the rotor with the mass of one and a half thousand tonnes which came off the axle and ruined the turbine and the main generator hall of Sayano–Shushenskaya hydroelectric power station (this fatal accident took place August 17, 2009, and claimed lives of 75 people).

Theory. Due to unavoidable manufacturing errors the centre of mass of the rotor is always displaced from the axis of rotation by some distance which is called eccentricity. At rapid rotation of asymmetric rotor sufficiently great periodic centrifugal forces are generated. If frequencies of these forces coincide with frequencies of characteristic vibrations of the construction, resonance effect is observed at which abrupt increase in the amplitude of forced vibrations occurs. There may be several resonant frequencies corresponding with different modes of vibration.

Task. In the given problem it is suggested to investigate behaviour of an asymmetric fan, fixed on a T-shaped support, at various rotational frequencies.

1. Take the dependence of frequency ν of fan rotation on the applied voltage U which shall not exceed $U_{\max} = 15$ V. Measure the threshold voltage $U_{\min}$ at which the rotation begins and the running frequency ν_0 at the nominal voltage $U_0 = 12$ V.
2. Determine resonant frequencies of vibrations of the asymmetric fan (to increase the eccentricity use a piece of plasticine).
3. Investigate qualitatively the dependence of resonant amplitude A of vibrations of the asymmetric fan on its eccentricity ε.
4. Take the dependence of amplitude A of vibrations of the asymmetric fan on the frequency ν of its rotation (resonant curve) nearby the maximal resonant frequency $\nu_{\max}$.
5. Determine the quality factor Q of the vibrating system investigated in the previous paragraph.

Note. During the experiment it is prohibited to change any mechanical parameters of the facility except for the dimensions and the position of a piece of attached plasticine used to generate artificial asymmetry and to vary eccentricity magnitude. If at extreme resonant vibrations the fan begins to operate unsteadily, the amplitude of vibrations should be decreased to perform correct measurements.

Equipment. Fan fixed on T-shaped support, toothpick fixed on fan body, stroboscopic tachometer, source of constant adjustable voltage, voltmeter, plasticine, paper scotch tape, graph paper, ruler.

Possible solutions

Junior league

Problem 1. Mechanical «grey box»

Pay attention to the fact that while pulling the rod A out of the «grey box» the rod B stays immobile, but while pulling out the rod B the rod A pulls into the «grey box». On account of this observation fulfil three series of measurements using the dynamometer and graph paper.

1. Plot dependence of displacement x_1 of the rod A on applied force F_1 (table 6, fig. 12).
2. Plot dependence of displacement x_2 of the rod B on applied force F_2 (table 7, fig. 13), the rod A being unfixed.
3. Plot dependence of displacement x_3 of the rod B on applied force F_3 (table 8, fig. 14), the rod A being fixed.

From angular coefficients of obtained three linear dependences determine effective deflection rates $K_1 = (1.00 \pm 0.05)$ N/cm, $K_2 = (0.50 \pm 0.03)$ N/cm and $K_3 = (0.67 \pm 0.03)$ N/cm, respectively.

Table 6

#	F_1, N	x_1, cm
1	0.5	0.7
2	1.0	1.1
3	1.5	1.5
4	2.0	2.0
5	2.5	2.5
6	3.0	3.1
7	3.5	3.6
8	4.0	4.2
9	4.5	4.6
10	5.0	5.2

Table 7

#	F_2, N	x_2, cm
1	0.5	1.0
2	1.0	1.9
3	1.5	3.0
4	2.0	4.1
5	2.5	5.1
6	3.0	5.8
7	3.5	6.8
8	4.0	7.9
9	4.5	9.0
10	5.0	10

Table 8

#	F_3, N	x_3, cm
1	0.5	0.9
2	1.0	1.5
[illegible]	1.5	2.3
[illegible]	2.0	3.0
[illegible]	2.5	3.8
[illegible]	3.0	4.4
7	3.5	5.4
8	4.0	6.2
9	4.5	6.9
10	5.0	7.3

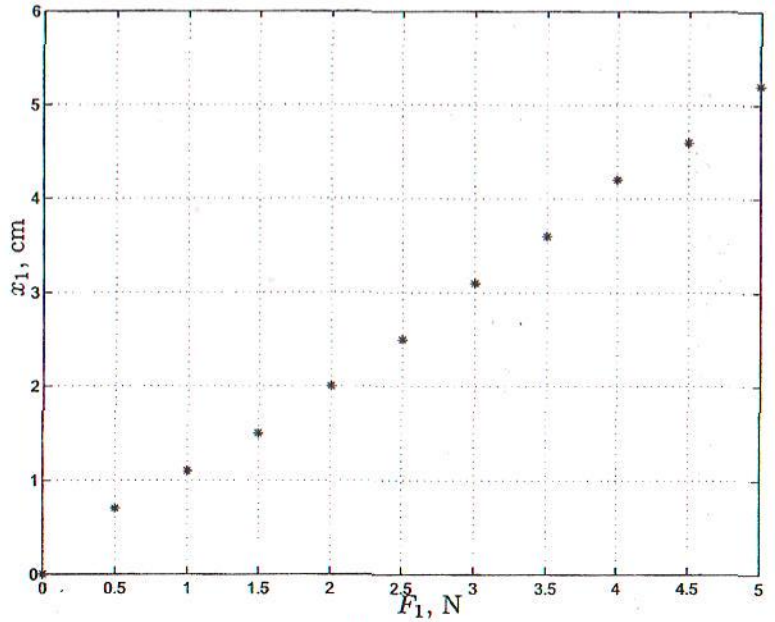


Fig. 12

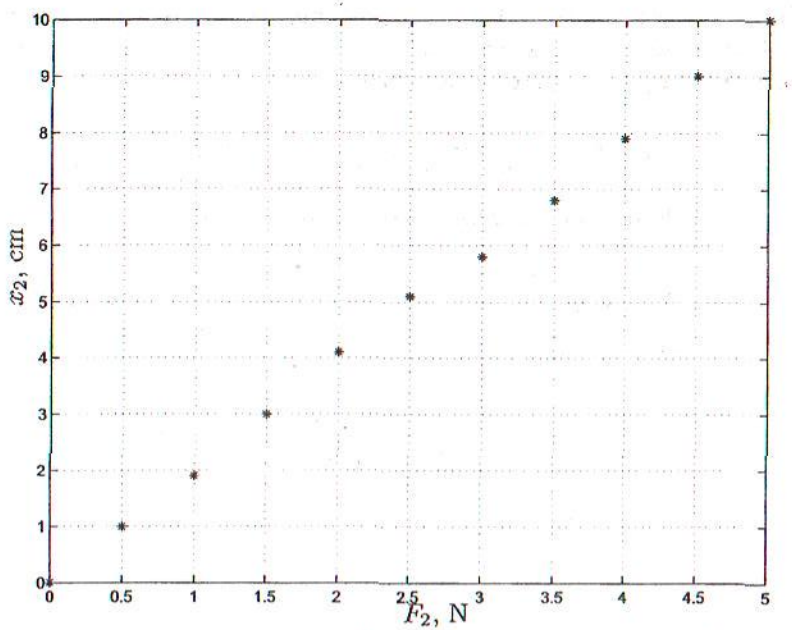


Fig. 13

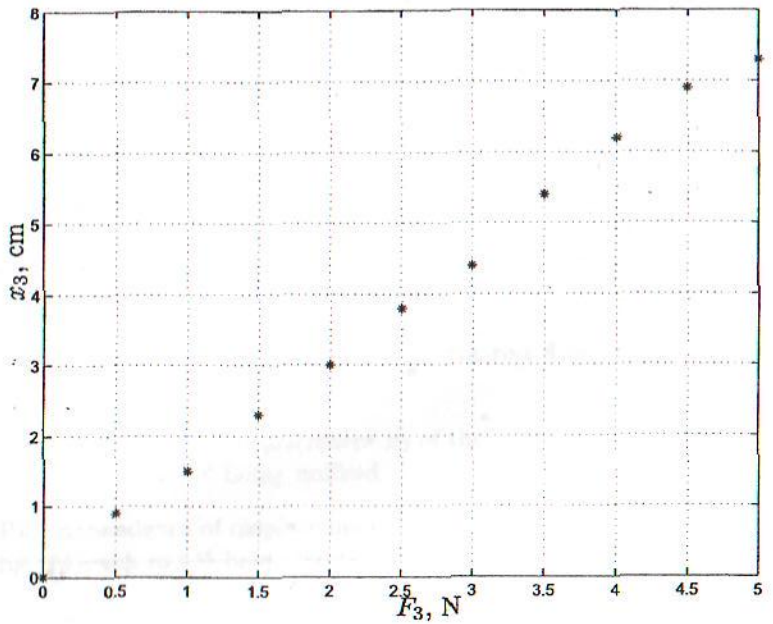


Fig. 14

On fig. 15 the simplest scheme corresponding to observed characteristics of the «grey box» is shown. Using formulas for total deflection rates of springs connected in series and in parallel write system of equations

$$K_1 = k_1, \qquad \frac{1}{K_2} = \frac{1}{k_2} + \frac{1}{k_3}, \qquad \frac{1}{K_3} = \frac{1}{k_1 + k_3} + \frac{1}{k_2}.$$

Subtract the third equation from the second one and substitute $k_1 = K_1$:

$$\frac{1}{K_2} - \frac{1}{K_3} = \frac{K_1}{k_3(K_1 + k_3)}, \qquad \text{whence} \qquad k_3^2 + K_1 k_3 - \frac{K_1 K_2 K_3}{K_3 - K_2} = 0.$$

Only the positive root has physical sense:

$$k_3 = \frac{K_1}{2}\left(\sqrt{1 + \frac{4K_2K_3}{K_1(K_3 - K_2)}} - 1\right).$$

From the second equation derive

$$k_2 = \frac{K_2 k_3}{k_3 - K_2}.$$

After substituting numerical values, errors being taken into account, finally obtain $k_1 = (1.00 \pm 0.05)$ N/cm, $k_2 = (1.0 \pm 0.1)$ N/cm, $k_3 = (1.0 \pm 0.2)$ N/cm.

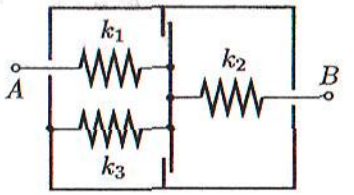


Fig. 15

The obtained results allow to conclude that when constructing boxes the identical springs have been used. Greater errors of k_2 and k_3 in comparison with k_1 result from more complicated formulas. The main contribution to the error of k_3 is made by the difference $(K_3 - K_2)$: absolute errors of the minuend and the subtrahend are $\Delta_3 = \Delta_2 = 0.03$ N/cm, the absolute error of the difference is $\Delta = \sqrt{\Delta_3^2 + \Delta_2^2} \approx$ ≈ 0.04 N/cm, the relative error of the difference is $\varepsilon = \Delta/(K_3 - K_2) \approx 0.25 = 25\%$. Other errors are small and a square root decreases the relative error by two times. By analogy, the main contribution to the error of k_2 is made by the difference $(k_3 - K_2)$.

Grading system

Tracing connection between rods movements 2
Table and plot for $x_1(F_1)$ having at least 4 points 1
Table and plot for $x_1(F_1)$ having at least 8 points 1
Table and plot for $x_2(F_2)$ having at least 4 points 1
Table and plot for $x_2(F_2)$ having at least 8 points 1
Table and plot for $x_3(F_3)$ having at least 4 points 1
Table and plot for $x_3(F_3)$ having at least 8 points 1
Scheme of «grey box» 3
Numerical answers for deflection rates of springs 3
Estimation of errors 1

Problem 2. «Black box» with three terminals

1. Make sure that there are no power sources within the black box.

2. We connect the battery to the terminals of the black box and measure the current intensity between its terminals. A constant current flows between the terminals 1 and 2, and no current flows between 1-3 and 2-3 for any polarity. We conclude that the scheme has the form of a star, one of the branches of which includes the capacitor. When changing the polarity between 1-2 the current intensity changes. This may be related to the fact that for one polarity the diode is closed, and for the other polarity the diode is open. Since the resistance of the open diode is much smaller than the resistance of the resistor, then the diode shunts it, which leads to an increase in the current intensity. From this result it follows that the diode is connected in parallel to one or both resistors (fig. 16).

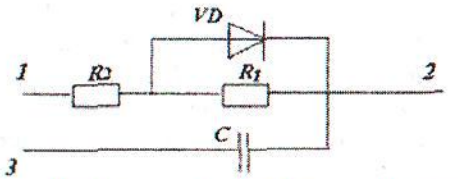


Fig. 16. Scheme of «black box»; $R_{22} =$ $= 2{,}2$ kΩ, $R_2 = 9{,}2$ kΩ

3. We measure the dependence U(I) for the given diode with the help of the variable resistor (fig. 17).

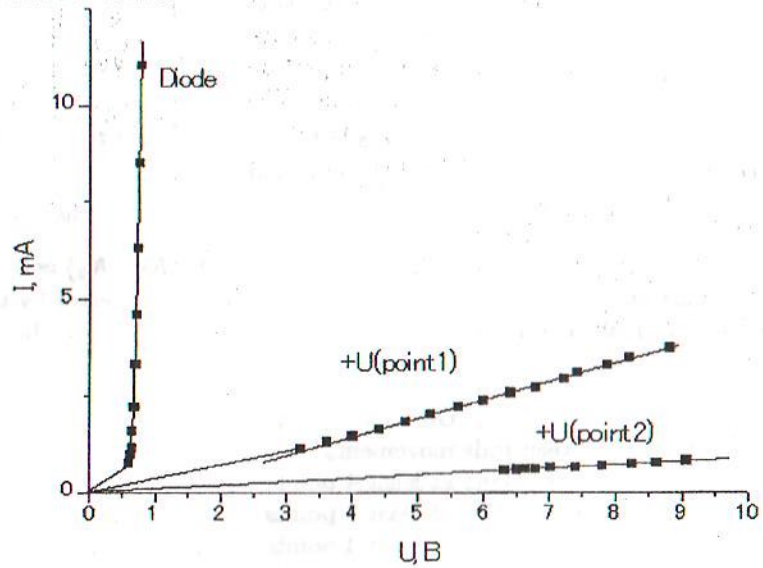


Fig. 17

4. Using the variable resistance, we construct volt-ampere characteristics (fig. 17) of the subcircuit between 1 and 2. The lowest curve is when «+» of the power supply is connected to point 2. In this case the diode is closed. From the volt-ampere characteristic (VAC) it is seen that the dependence is linear and passes through the origin of coordinates. From Ohm's Law we define the resistance $R_{12} \approx 11.4$ kΩ.

5. The middle line (fig. 17) is obtained when «+» of the power supply is connected to point 1. It is seen that this dependence is different from the VAC of the diode. We conclude that the diode is connected in series to R_2. When $U > 1$ V the resistance of the subcircuit is defined by R_2. Therefore, the resistance should be less than R_{12} and the current, respectively, more. From the slope on the subcircuit we determine the resistance $R_1 = \Delta U/\Delta I \approx 9.2$ kΩ.

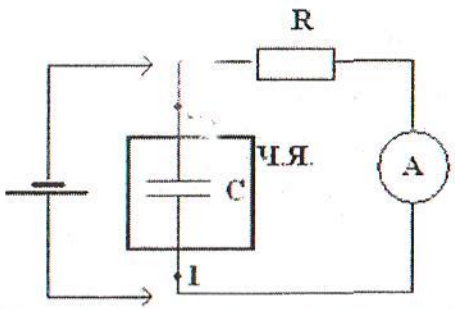


Fig. 18. Scheme for determining of capacitance of capacitor; $R = 340$ kΩ

6. We determine the resistance $R_1 = R_{12} - R_2 = 2.2$ kΩ.

7. Determining the capacitor capacitance. We connect up the scheme fig. 18. Both terminal 1 and terminal 2 of the black box can be used. We charge the capacitor from the power supply. Then we disconnect it and, using the stopwatch, we measure the dependence $I(t)$ (fig. 19). From the formula $I = I_0 \exp(-t/RC)$

we define the capacitance $C = \tau/R$, where τ is the time during which I is reduced in e times.

Note. Ratings of the elements in the mentioned solution may be different from those used in the assembly of the black box. •

Note. The problem is included in the kit in the author's version. •

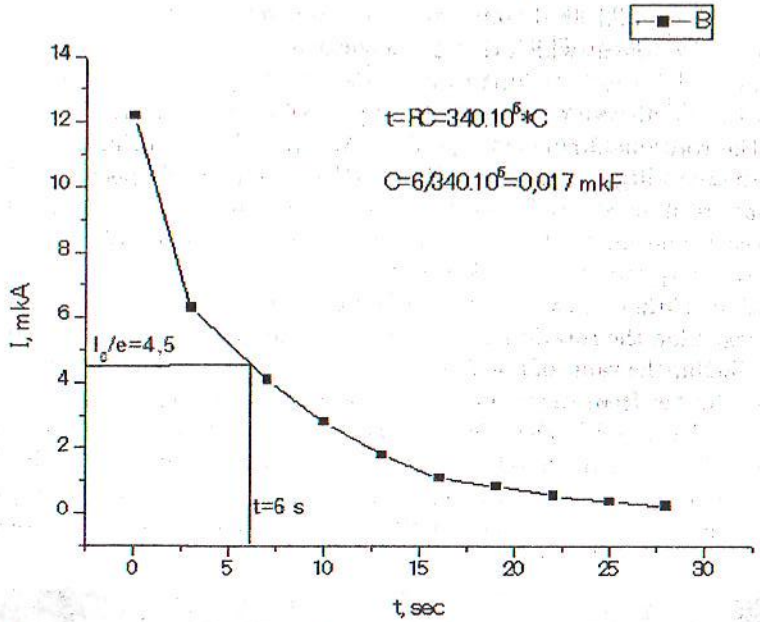


Fig. 19. Dependence $I(t)$ of circuit of discharge of capacitor

Senior league

Problem 1. Eccentric fan

The experimental equipment is shown on the photo (fig. 20).

1. Mark one of fan blades with a small piece of scotch tape. Apply to the fan the voltage U and gradually increase it from zero to $U_{\max} = 15$ V. Detect the threshold voltage at which the rotation begins: $U_{\min} \approx 4$ V. To vary the frequency of fan rotation adjust the frequency ν of stroboscopic tachometer flickering in such a way that blade marked with scotch tape looks in light flashes like being immobile, then the rotational frequency equals ν. Accuracy of measuring ν may be increased if one uses multiple frequencies, but in this case it is not necessary since error of ultimate result is essentially due to other factors. Measurement results are presented in table 9 and on the diagram (fig. 21). At the nominal voltage $U_0 = 12$ V the operating frequency is $\nu_0 = 2548$ rpm ≈ 42.5 Hz.

Note. If the frequency of flashes is adjusted to be by the integer number n of times less than the rotational frequency, the marked blade will also look like being immobile but the value of ν will be incorrect. To avoid this mistake one should make sure that as the frequency ν increases by any integer number n of times the number of observed marked blades also increases by n times. It is sufficient to choose as n several initial prime numbers (2, 3, 5, 7) because 11(and more)-fold error may be identified by means of estimating expected order of measured frequency. Since ν is continuous and monotone function of U one accomplishment of described test procedure will be enough.

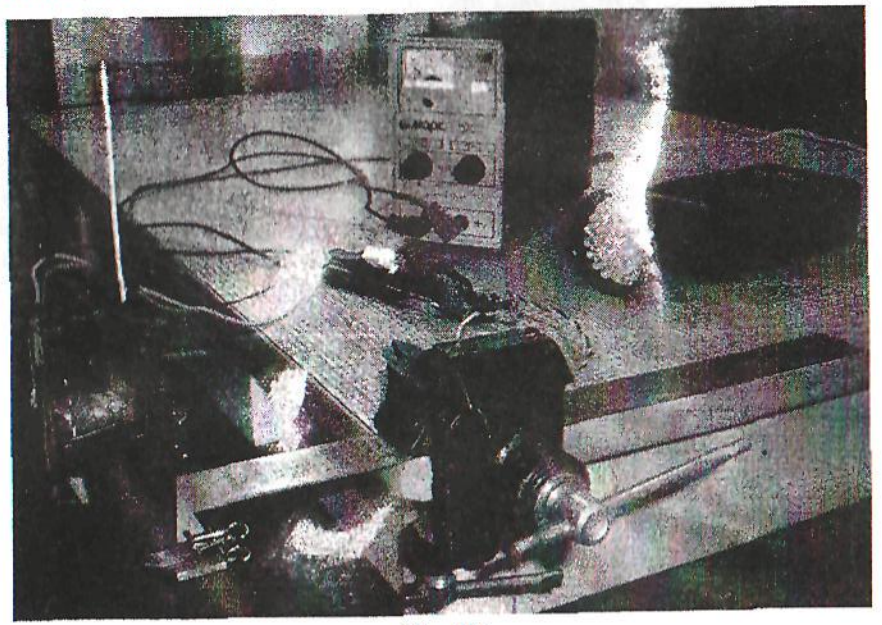


Fig. 20

Table 9

U, V	ν, rpm	ν, Hz
0	0	0
1	0	0
2	0	0
3	0	0
4	0	0
5	721.4	12.0
6	900.6	15.0
7	1160	19.3
8	1512	25.2
9	1784	29.7
10	2060	34.3
11	2297	38.3
12	2548	42.5
13	2782	46.4
14	3014	50.2
15	3229	53.8

Table 10

ν, rpm	ν, Hz	$A/A_{\max}$
1122	18.7	0.05
1278	21.3	0.05
1368	22.8	0.09
1464	24.4	0.18
1500	25.0	0.30
1518	25.3	0.48
1523	25.4	0.61
1527	25.5	0.68
1530	25.5	0.75
1532	25.5	0.91
1540	25.7	1.00
1620	27.0	0.25
1668	27.8	0.18
1753	29.2	0.11
1800	30.0	0.09
1848	30.8	0.09

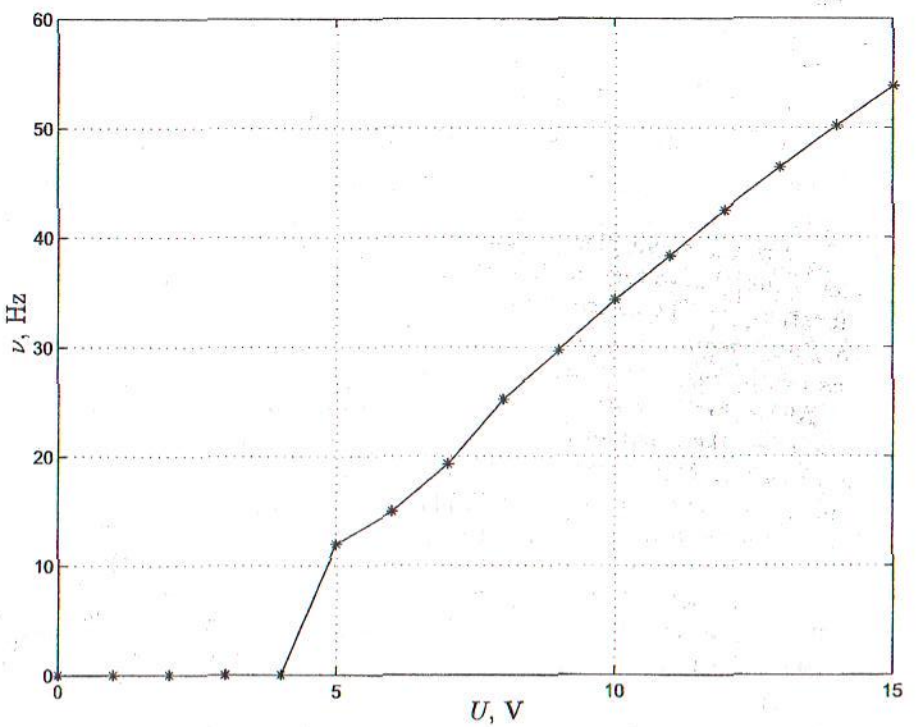


Fig. 21

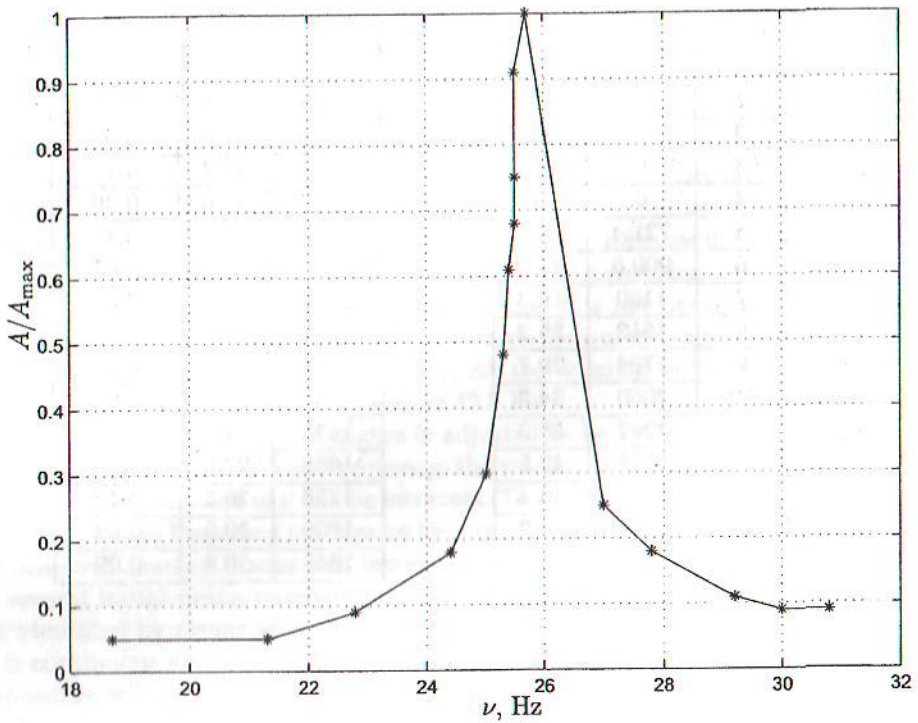


Fig. 22

2. Fix the small piece of plasticine on the edge of one of fan blades. Slowly increasing the voltage and, correspondingly, the rotation frequency, evaluate vibrations by the range R (double amplitude) of vibrations of the end of the toothpick mounted to the fan body. Notice that the fan and the T-shaped support has two resonant frequencies.

At the first vibration mode the fan body displaces up and down and the ruler bends. At the resonant frequency $\nu_1 = 900$ rpm ≈ 15 Hz the range $R_1 \approx 14$ mm is observed. At the second vibration mode the fan body rocks from side to side and the ruler twists. At the resonant frequency $\nu_2 = 1560$ rpm ≈ 26 Hz the range $R_2 \approx 15$ mm is observed.

3. Varying the mass m of pitched plasticine and distance r from the plasticine to the axle, notice that the resonant amplitude A is proportional to m and r, from which it follows that $A \sim \varepsilon$ since the eccentricity ε is proportional to mr.

4. Take the dependence of amplitude A of toothpick end vibrations on the frequency ν of asymmetric fan rotation nearby the second resonant frequency. For illustrative purposes use the reduced amplitude A/A_{max} instead of initial values A. Measurement results are presented in table 10 and on the diagram (fig. 22).

5. Using the graph of resonant curve (fig. 22) determine the central frequency ν_{res} and the width $\Delta\nu_{res}$ at the level $1/\sqrt{2} \approx 0.7$ of the maximum: $\nu_{res} \approx 26$ Hz

and $\Delta\nu \approx 0.8$ Hz. Calculate the required quality factor of vibrations according to formula

$$Q = \frac{\nu_{res}}{\Delta\nu_{res}} \approx 33.$$

Grading system

Method of measuring rotational frequency	2
Table of values U and ν	1
Plot of dependence $\nu(U)$	1
Numerical values of U_{min} and ν_0	1
Detecting exactly two resonant modes	1
Resonant frequency ν_1	1
Resonant frequency ν_2	1
Analysing dependence of A on m and r	2
Conclusion of proportionality of A and ε	1
Table of values ν and A	1
Plot of dependence $A(\nu)$	2
Numerical value of Q	1