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Физико-математический форум "Ленский край"
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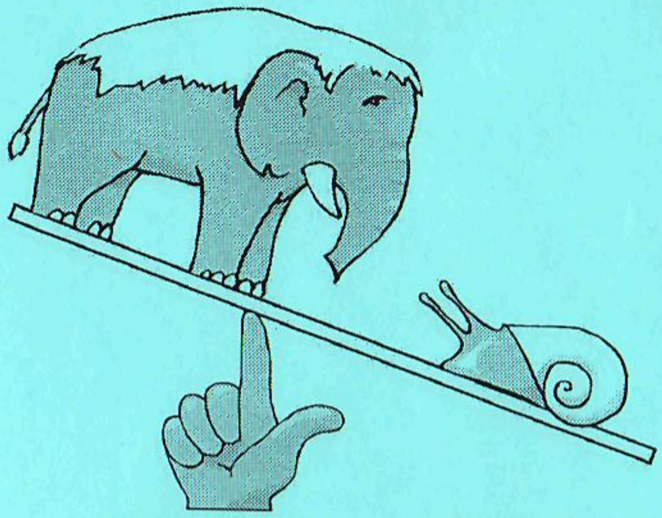
XI Международная физическая олимпиада "Туймаада"
XI International physics olympiad "Tuymaada"

Физика



Якутск, 7–15 июля 2004 г.

Problem 1 (Wheel)

A wheel of radius R, agglutinated from two homogeneous halves of masses m_1 and m_2 is placed on a rough horizontal surface (Fig. 1). Answer the following questions:

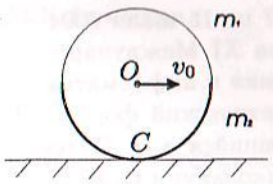


Fig. 1

1. What is the speed v_0 of the center O for the wheel to make a full turn ?
2. Determine the period of small oscillations of the wheel near the equilibrium position.
3. Find the maximum inclination angle α_{max} of the surface to the horizon at which the wheel is still remaining in equilibrium on the surface.

Problem 2 (Hall effect)

A semiconductor sample is placed into the magnetic field of induction $\vec{B}$. A current I flows through the sample (Fig. 2). The sample has the shape of a rectangular plate of thickness a. A potential difference U emerges between A and B, at that $U = R\frac{BI}{a}$, where R is the Hall's coefficient.

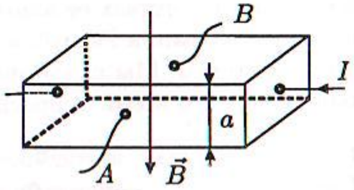


Fig. 2

1. Taking into account that the conductivity of the sample is formed by both electrons and holes, determine R. The holes are particles which possesses a positive charge equal by value to the charge of an electron. The concentrations and mobilities of electrons and holes are n, p, μ_n and μ_p respectively. The mobility is a ratio of the velocity of charge carrier v to field strength E, i.e $v = \mu E$.

Problem 3 (Thermodynamics cycle)

One mole of an ideal gas undergoes the cycle shown in figure 3 , where 1–2 is an isotherm, 2–3 is an isobar, 3–4 is an polytrope for which $C = R/2$ and 4–1 is an isochor. The minimum reached temperature during the cycle equals $T_{min} = 300$ K. The polytropic process is a process at constant heat capacity C. Answer the following questions:

1. Indicate the points on the cycle at which the gas reaches the maximum T_{max} and the minimum T_{min} temperatures and determine T_{max}.

2. What is the net heat Q_+ added to the gas during the cycle?
3. What is the work A of the gas during the cycle?
4. Determine the efficiency η of the cycle and compare it with the efficiency of an ideal heat engine, which operates between the heater and the cooler with temperatures T_{max} and T_{min} respectively.

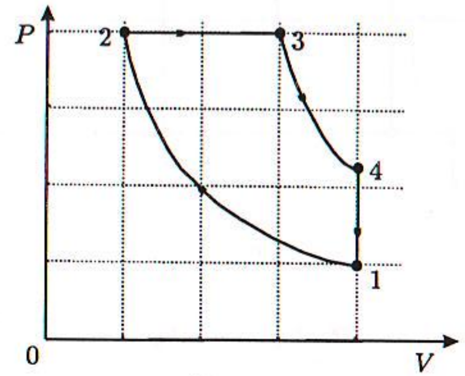


Fig. 3

Problem 4 (Aquarium)

In this problem you will consider some optical properties of system aquarium.

1. The wall of the glass aquarium of radius $R = 10$ cm and thickness $\delta = 5$ mm has a refrection index $n = 1.6$. Determine the focal distance R_f of the splinter of the aquarium.
2. Let now the thickness of the wall equals $\frac{R}{2}$. Determine the focal distance F of the "thick"aquarium.
3. A point light source is placed at the distance $2R$ from the center of the "thick"aquarium. An observer located on the other side of the aquarium sees an image of the source at the distance s from the external side of the aquarium. Determine s.

Problem 1 (Semiinfinite chain)

Determine the resistance of the semiinfinite chain between distinguished two points (Fig. 4) if the resistance of each section equals r.

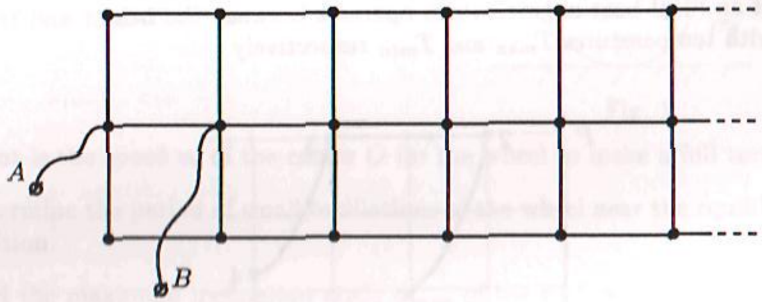


Fig. 4

Problem 2 (Puck)

A cylindrical puck of radius R is sliding without friction on the horizontal icy surface with velocity v_0, spinning around the axis of symmetry with angular velocity ω_0. The puck strikes a vertical wall at angle φ. After the elastic collision it has velocity v at angle ψ to the wall and spins with angular velocity ω (Fig. 5). The wall's constant of friction equals μ.

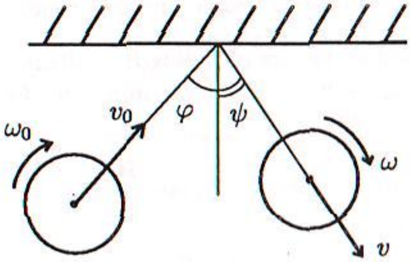


Fig. 5

Find the expressions for v, ω, ψ in terms of v_0, ω_0, φ, R, μ when $\omega_0 > \frac{4\mu v_0 \cos\varphi}{R}$. What are the values of μ for the puck to be rebounded perpendicular to the wall, in the opposite direction, without spinning after the collision?

Problem 3 (Process)

One mole of an ideal undergoes the process shown in figure 6. Determine the maximum temperature of the gas during the cycle (One may assume that the process is quasistatic).

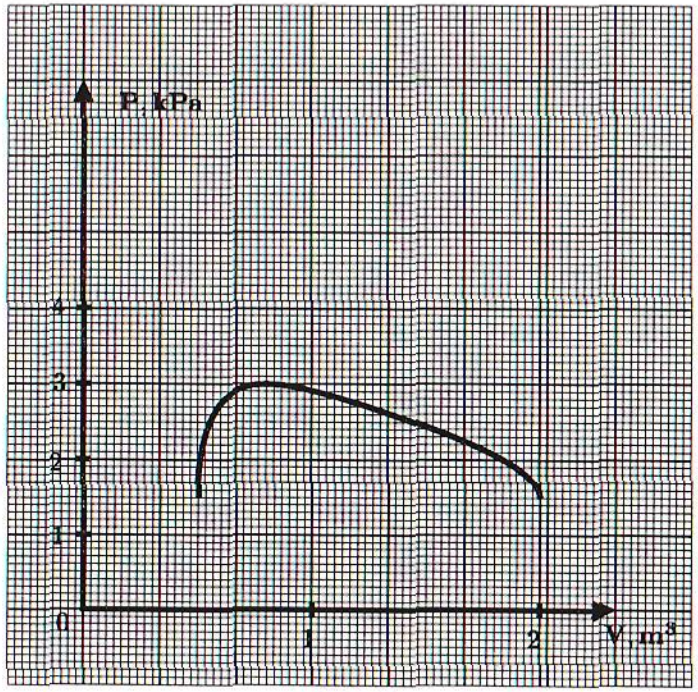


Fig. 6

Problem 4 (Aquarium)

The wall of the glass aquarium of radius $R = 10$ cm and thickness $\delta = 5$ mm has a refrection index $n = 1.6$. Determine the focal distance R_f of the splinter of the aquarium.

High league
Problem1 (Wheel)

The initial kinetic energy of the wheel may be expressed as:

$$E_k = \frac{I_c w^2}{2} \quad (1.0 \text{ point}),$$

where I_c is the moment of inertia of the wheel about C and $w = v_0/R$ is the angular velocity of the rotating wheel. The position of wheel's center of gravity is given by (Fig. 13):

$$y_0 = \frac{\int_0^{\pi/2} \frac{m}{\pi R} R \sin\varphi \, 2R\,d\varphi}{m} = \frac{2R}{\pi} \quad (0.5 \text{ points}).$$

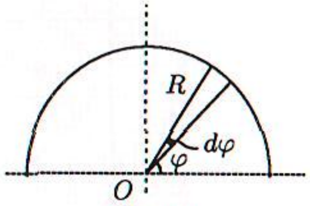


Fig. 13

From Steiner's theorem the moment of inertia about C may be written as:

$$I_c = I_1 + I_2 =$$

$$= (m_1+m_2)R^2\left(1-\frac{4}{\pi^2}\right) + m_2R^2\left(1-\frac{2}{\pi}\right)^2 + m_1R^2\left(1+\frac{2}{\pi}\right)^2 =$$

$$= 2R^2\left(m_1\left[1+\frac{2}{\pi}\right] + m_2\left[1-\frac{2}{\pi}\right]\right) \quad (2.0 \text{ points}).$$

Where I_1 and I_2 are the moments of inertia of upper and lower halves of the wheel respectively. The maximum potential energy of the wheel reached during the motion is:

$$\Delta\Pi = (m_1+m_2)g\Delta h = 4(m_1+m_2)g\frac{m_2-m_1}{m_2+m_1}\frac{R}{\pi} = 4(m_2-m_1)g\frac{R}{\pi} \quad (1.0 \text{ point}).$$

Equalizing changes in potential and kinetic energies we obtain:

$$\left(1+\frac{2}{\pi}\right)m_1v_0^2 + \left(1-\frac{2}{\pi}\right)m_2v_0^2 = 4(m_2-m_1)g\frac{R}{\pi},$$

$$v_0 = \sqrt{\frac{4(m_2-m_1)gR}{m_1(\pi+2)+m_2(\pi-2)}} \quad (1.0 \text{ point}).$$

Let's get an oscillation period of the wheel. A small displacement on angle φ from the equilibrium state is produced. The values for the kinetic and potential energies during the motion are written as follows.

$$\Delta\Pi = (m_1+m_2)\frac{m_2-m_1}{m_2+m_1}g\frac{2R}{\pi}(1-cos\varphi) \approx (m_2-m_1)g\frac{R}{\pi}\varphi^2 \quad (1.0 \text{ point}).$$

Kinetic energy of the system equals to $K = \frac{I_c\dot{\varphi}^2}{2}$ (1.0 point), where I_c is the moment of inertia of the wheel about the instantaneous axis. The angle φ being changed the value of I_c also changes, but due to the smallness of the oscillations these changes could be ignored. Since:

$$I_c = 2R^2\left(m_1\left[1+\frac{2}{\pi}\right] + m_2\left[1-\frac{2}{\pi}\right]\right),$$

the period of small oscillations is:

$$T = 2\pi\sqrt{\frac{2R^2\left(m_1\left[1+\frac{2}{\pi}\right]+m_2\left[1-\frac{2}{\pi}\right]\right)}{2\frac{R}{\pi}(m_2-m_1)g}} = 2\pi\sqrt{\frac{R}{g}}\sqrt{\pi\frac{m_2+m_1}{m_2-m_1}-2} \quad (1.0 \text{ point}).$$

When the inclination angle of the surface reaches its maximum value α_{max}, the line of gravity force applied to the center of masses M, passes through the point C (Fig. 14) (1.0 point). This is because the net moment of forces about C is zero. From the geometrical considerations we yield:

$$\alpha_{max} = \arcsin\frac{2}{\pi}\left(\frac{m_2-m_1}{m_2+m_1}\right) \quad (1.0 \text{ point}).$$

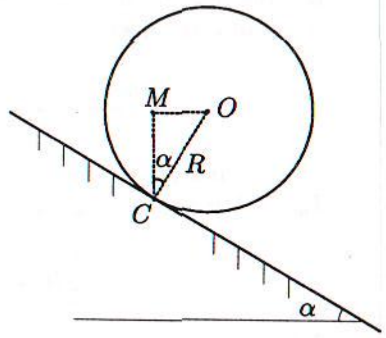


Fig. 14

Problem 2 (Hall effect)

Let's consider a semiconductor where a motion of electrons and holes exists. A net current equals $I = eE_{\parallel}(n\mu_n + p\mu_p)ab$ (2.0 points) where b is the sample width, $E_{\parallel}$ is the longitudinal component of the field strength in the semiconductor, e is an absolute value of the carrier's charge. The longitudinal velocities of holes and electrons respectively are $v_{\parallel}^p = \mu_p E_{\parallel}$ and $v_{\parallel}^n = \mu_n E_{\parallel}$. The force acting on a hole in the lateral direction is given by the expression $F_{\perp}^p = eE_{\perp} + ev_{\parallel}^p B = eE_{\perp} + e\mu_p E_{\parallel} B$ (1.0 point). Similarly for an electron $F_{\perp}^n = e\mu_n E_{\parallel} B - eE_{\perp}$ (1.0 point), where $E_{\perp}$ is the lateral component of the electric field. Hence:

$$v_{\perp}^p = \mu_p E_{\perp} + \mu_p^2 E_{\parallel} B \qquad v_{\perp}^n = \mu_n^2 E_{\parallel} B - \mu_n E_{\perp} \quad (1.0 \text{ point}).$$

The fluxes of electrons and holes must compensate each other in the steady state $epv_{\perp}^p = env_{\perp}^n$ (3.0 points). Using the above results we find an expression for $E_{\perp}$:

$$p\mu_p(\mu_p E_{\parallel} B + E_{\perp}) = n\mu_n(\mu_n E_{\parallel} B - E_{\perp}) \quad \Rightarrow E_{\perp} = \frac{n\mu_n^2 - p\mu_p^2}{n\mu_n + p\mu_p}E_{\parallel}B \quad (1.0 \text{ point}).$$

Substituting the expression for $E_{\parallel}$ we find:

$$E_{\perp} = \frac{n\mu_n^2 - p\mu_p^2}{(n\mu_n + p\mu_p)^2}\frac{IB}{eab} \quad \text{and } R = \frac{n\mu_n^2 - p\mu_p^2}{e(n\mu_n + p\mu_p)^2} \quad (1.0 \text{ point}).$$

Problem 3 (Thermodynamics cycle)

Obviously the temperature of the gas reaches its minimum value on the isotherm 1–2 (0.5 points). During the process 2–3 the gas is heated since it produces a positive work, and during 3–4 it is cooled due to $C_V > C$. Therefore, the gas reaches the maximum temperature at 3 (0.5 points). By the gaseous state equation, we have:

$$T_{max} = T_{min}\frac{V_3}{V_2} = 3T_{min} = 900 \text{ K} \quad (0.5 \text{ points}).$$

Heat is added to the gas only during 2–3, thus $Q_+ = C_P(T_3 - T_2) = C_P(T_{max} - T_{min})$. For the monoatomic gas:

$$C_P = \frac{5}{2}R \Rightarrow Q_+ = \frac{5}{2}R(T_{max} - T_{min}) \quad (1.0 \text{ point}).$$

Substituting the numerical data we obtain $Q_+ = 12.5$ kJ (0.5 points).

For the work we have $A = A_{23} + A_{34} - A_{21}$. Since 2–3 is an isobar then $A_{23} = p_2(V_3 - V_2) = R(T_3 - T_2) = R(T_{max} - T_{min})$ (0.5 points). The work during the polytropic process is found according to the second law of thermodynamics $C(T_4 - T_3) = C_V(T_4 - T_3) + A_{34}$ или $(C_V - C)(T_{max} - T_4) = A_{34}$ (0.5 points). For precise determination of T_4 an equation for the polytrope is found: $pdV = (C - C_V)dT$ taking into account that $pdV + V\,dp = RdT$ we obtain:

$$pdV = \frac{C - C_V}{R}(pdV + V\,dp) \Rightarrow$$

$$\int \frac{dV}{V} = \frac{C - C_V}{C_p - C}\int \frac{dp}{p} \Rightarrow pV^{\frac{C - C_p}{C - C_V}} = const \quad (1.0 \text{ point}).$$

Taking into consideration that $C = R/2$ the equation of the polytrope is obtained $pV^2 = RTV = const$. From the plot of the cycle:

$$\frac{V_3}{V_4} = \frac{3}{4}, \text{ thus } T_4 = \frac{3}{4}T_3 = \frac{3}{4}T_{max} \text{ and}$$

$$A_{34} = (C_V - C)\frac{1}{4}T_{max} = \frac{RT_{max}}{4} \quad (1.0 \text{ point}).$$

The work on the isotherm is left to be determined.

$$dA_{21} = pdV = RT_{min}\frac{dV}{V} \Rightarrow A_{21} = RT_{min}\int_{V_2}^{V_1}\frac{dV}{V} = RT_{min}\ln\frac{V_1}{V_2} \quad (1.0 \text{ point}).$$

Because of $\frac{V_1}{V_2} = 4$ then $A_{21} = RT_{min}\ln 4$ (0.5 points). Eventually:

$$A = R(T_{max} - T_{min}) + \frac{RT_{max}}{4} - RT_{min}\ln 4 \quad (0.5 \text{ points}).$$

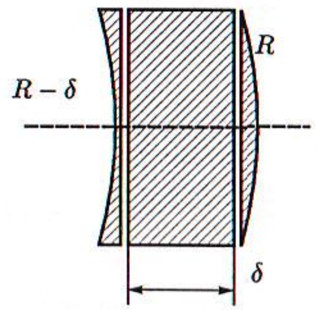
Substituting the numerical data we obtain $A = 3.40$ kJ (0.5 points).

Efficiency of the cycle is calculated by the formula $\eta = \frac{A}{Q_+}$ (0.5 points). Substituting the numerical data $\eta = 27\%$ (0.5 points). For an ideal heat Carnot engine we have:

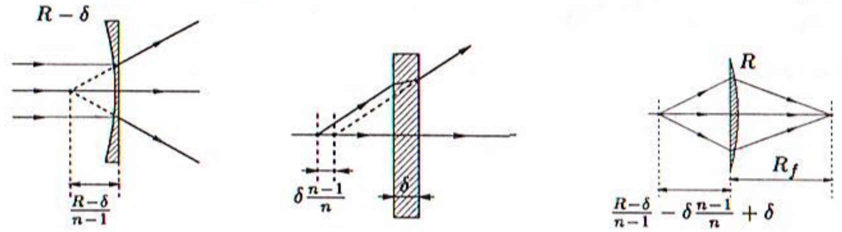
$$\eta_0 = \frac{T_{max} - T_{min}}{T_{max}} = 67\% \quad (0.5 \text{ points}).$$

Problem 4 (Aquarium)

The splinter is equivalent to the scheme (1.0 point):



Now the images created by the constituents of the scheme are found (0.5 points):



From the formula of the lens:

$$\frac{1}{\frac{R-\delta}{n-1}-\delta\frac{n-1}{n}+\delta}+\frac{1}{R_f}=\frac{n-1}{R} \quad (0.5 \text{ points}).$$

From this formula, neglecting the higher infinitesimal orders of δ , we obtain:

$$\frac{1}{R_f}=-\frac{n-1}{n}\frac{\delta}{R^2} \quad (0.5 \text{ points}),$$

hence:

$$R_f=-\frac{n}{n-1}\frac{R^2}{\delta}\approx -5.3 \text{ m} \quad (0.5 \text{ points}).$$

The focal distance of the "thick"aquarium will be found now. Let's consider refractions on the four surfaces of the beam incident along the optical axis (Fig. 15). Refraction on the first surface 1 with curvature radius R:

$$\frac{n}{x_1}=\frac{n-1}{R}\Rightarrow x_1=\frac{n}{n-1}R\Rightarrow x_2=x_1-\frac{R}{2}=\frac{n+1}{2(n-1)}R \quad (1.0 \text{ point}).$$

Refraction on the second surface 2 with curvature radius $\frac{R}{2}$:

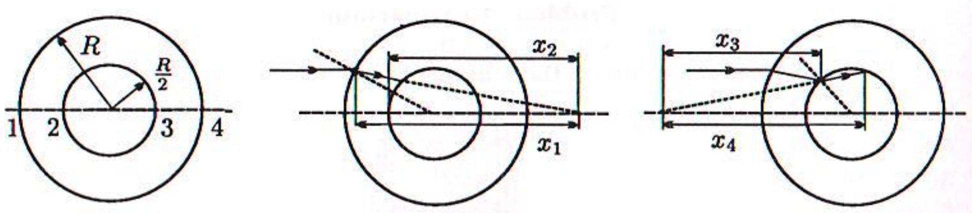


Fig. 15

$$-\frac{n}{x_2}+\frac{1}{x_3}=\frac{1-n}{R/2}\Rightarrow x_3=-\frac{R}{2}\frac{n+1}{n-1}\Rightarrow x_4=-x_3+R=\frac{3n-1}{2(n-1)}R \quad (1.0 \text{ point}).$$

Refraction on the third surface 3 with curvature radius $\frac{R}{2}$:

$$\frac{1}{x_4}+\frac{n}{x_5}=\frac{n-1}{-R/2}\Rightarrow x_5=\frac{3n-1}{6(1-n)}R\Rightarrow x_6=-x_5+\frac{R}{2}=\frac{3n-2}{3(n-1)}R \quad (1.0 \text{ point}).$$

Refraction on the fourth surface 4 with curvature radius R:

$$\frac{n}{x_6}+\frac{1}{x_7}=\frac{1-n}{-R}=\frac{n-1}{R}\Rightarrow x_7=\frac{3n-2}{2(1-n)}R \quad (1.0 \text{ point}).$$

Finally for the focal distance:

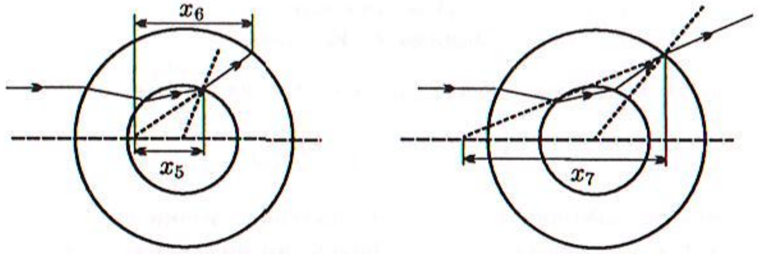


Fig. 15

$$F=-\frac{3n-2}{2(n-1)}R\approx -23.3 \text{ cm} \quad (1.0 \text{ point}).$$

To determine the image position S' a beam through the optical center of the aquarium O and a beam parallel to the optical center are drawn (Fig. 20). Their intersection points out the image position (0.5 points). Let h be the distance between the incident beam and the axis, then at the output it equals:

$$h_x=h\cdot\frac{x_2}{x_1}\cdot\frac{x_4}{x_3}\cdot\frac{x_6}{x_5}=h\left(3-\frac{2}{n}\right) \quad (0.5 \text{ points}).$$

From the triangle formed by the beams and the optical axis we get:

$$s=\frac{7n-4}{5n-4}R=18 \text{ cm} \quad (0.5{+}0.5 \text{ points}).$$

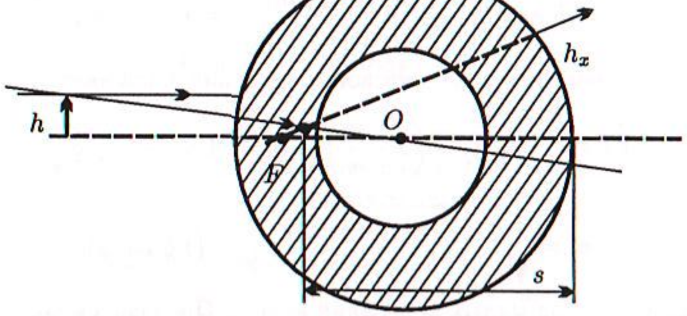


Fig. 16

First league
Problem 1 (Semiinfinite chain)

The potentials of all symmetrical to AB points equals due to the symmetry of the chain with respect to the line connecting points A and B (3.0 points). Consequently, the schemes in Fig. 4 and Fig. 21 are equivalent (2.0 points). It is easy to calculate the resistance of the scheme in Fig. 21 now:

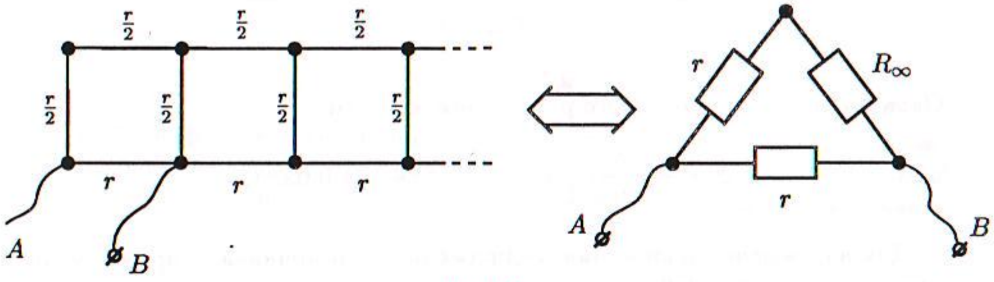


Fig. 21

$$R_\infty = \frac{\frac{r}{2}(R_\infty + \frac{3}{2}r)}{R_\infty + \frac{3}{2}r + \frac{r}{2}} \quad \text{(3.0 points)},$$

whence $R_\infty = \dfrac{r}{4}$. Hence:

$$R_{AB} = \frac{5}{9}r \quad \text{(2.0 points)}.$$

Problem 2 (Puck)

The frictional and normal reaction forces are acting on the puck during the collision (Fig. 22):

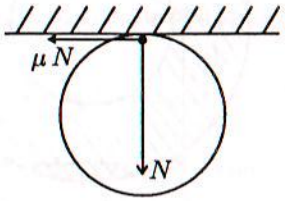


Fig. 22

Surely N will vary with time in a complicated manner , but it's important that the frictional force is μ times greater than N during the whole collision. The reaction force changes the impulse of the puck along y on $2mv_0\cos\varphi$, therefore the frictional force changes the impulse of the puck along x on $2\mu\, mv_0\cos\varphi$.

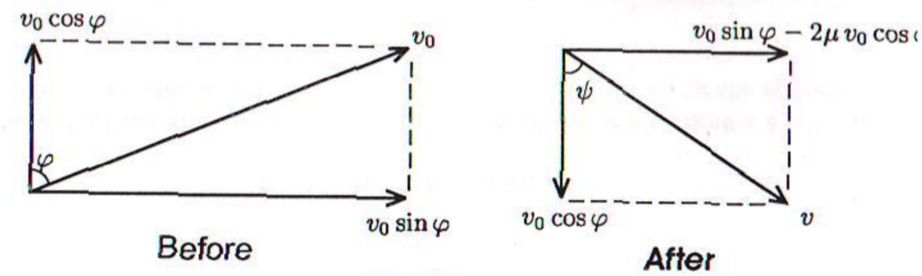


Fig. 23

From Fig. 23 we see:

$$\operatorname{tg}\psi = \frac{v_0\sin\varphi - 2\mu\, v_0\cos\varphi}{v_0\cos\varphi} = \operatorname{tg}\varphi - 2\mu \quad \text{(3.0 points)},$$

$$v = \sqrt{(v_0\cos\varphi)^2 + (v_0\sin\varphi - 2\mu\, v_0\cos\varphi)^2},$$

$$v = v_0\sqrt{1 + 4\mu\cos\varphi(\mu\cos\varphi - \sin\varphi)} \quad \text{(1.0 point)}.$$

The puck rebounds perpendicular to the wall in case $\psi = 0$, i.e when $\mu = \operatorname{tg}\varphi/2$ (1.0 point). The puck rebounds in the opposite direction in case $\psi = -\varphi$, i.e when $\mu = \operatorname{tg}\varphi$ (1.0 point). $\vec{L}$ is an the angular moment of the puck relative to the axis of the symmetry, $\vec{M}$ is the momentum of the frictional forces, I is the moment of inertia of the puck, τ is the collision's duration. From the momental equation:

$$\frac{d\vec{L}}{dt} = \vec{M},$$

$$\int_{I\omega_0}^{I\omega} = -\mu R\int_0^\tau N\,dt = \mu R\, 2mv_0\cos\varphi,$$

$$I(\omega - \omega_0) = -2\mu\, mRv_0\cos\varphi,$$

$$\frac{1}{2}mR^2(\omega_0 - \omega) = 2\mu\, mRv_0\cos\varphi,$$

$$\omega = \omega_0 - \frac{4\mu\, v_0\cos\varphi}{R} \quad \text{(3.0 points)}.$$

The puck won't spin after the collision in case $\omega = 0$, i.e when:

$$\mu = \frac{\omega_0 R}{4v_0\cos\varphi} \quad \text{(1.0 point)}.$$

Problem 3 (Process)

The process is gigen in P–V axes. Let's plot the process in p–$\frac{1}{V}$ axes (Fig. 24):

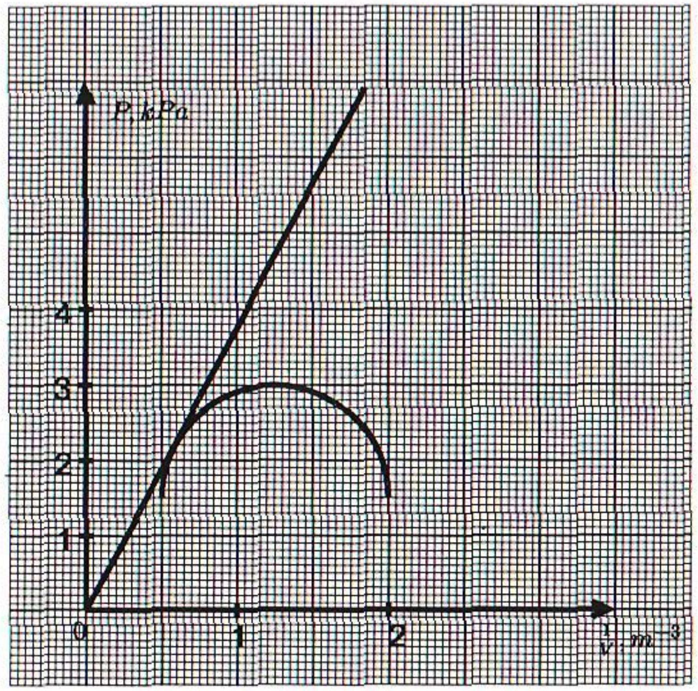


Fig. 24

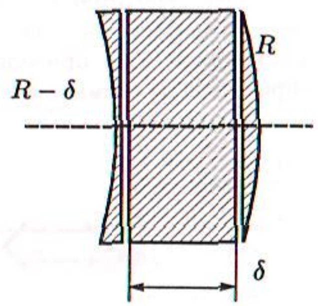
The isotherms are lines on p–$1/V$ diagram, which pass through the origin point. The tangent of the isotherm's inclination angle is directly proportional to the temperature. Now the line which passes through the origin point and has the maximum inclination angle and has at least one general point with the process graph is found (5.0 points). This is the very isotherm to be correspondent to the maximum temperature during the process. By the gaseous state equation, we have:

$$PV = \nu RT \Rightarrow P = \frac{1}{V}\nu RT \Rightarrow T_{max} = \frac{\operatorname{tg}\alpha}{\nu R} \quad \text{(4.0 points)}$$

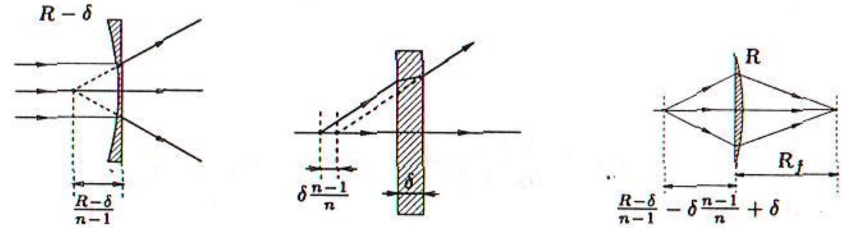
From the graphical measurement we obtain $\operatorname{tg}\alpha_{max}$ therefore $T_{max} \approx 450K$ (1.0 point).

Problem 4 (Aquarium)

The splinter is equivalent to the scheme (3.0 points):



Now the images created by the constituents of the scheme are found (3.0 points):



From the formula of the lens:

$$\frac{1}{\frac{R-\delta}{n-1} - \delta\frac{n-1}{n} + \delta} + \frac{1}{R_f} = \frac{n-1}{R} \quad \text{(2.0 points).}$$

From this formula, neglecting the higher infinitesimal orders of δ , we obtain:

$$\frac{1}{R_f} = -\frac{n-1}{n}\frac{\delta}{R^2} \quad \text{(1.0 point),}$$

hence:

$$R_f = -\frac{n}{n-1}\frac{R^2}{\delta} \approx -5.3\ \text{m} \quad \text{(1.0 point).}$$