Министерство образования Республики Саха (Якутия)
Северо-Восточный федеральный университет им. М.К. Аммосова
Малая академия наук Республики Саха (Якутия)

Физика.
Экспериментальный тур
Методическое пособие

Якутск, 15-22 июля, 2017 г.
waves. A degree of polarization $P$ of the partially polarized light is determined by a formula

$$P = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}$$  \quad (1)$$

where $I_{\text{max}}$ and $I_{\text{min}}$ are maximum and minimum values of light intensity.

A polarizer is a device that lets the oscillations of the vector of electric field strength pass lying in only one plane.

Fig. 2

When the electromagnetic wave falls on a dielectric, the oscillations of charged particles (electrons) arise in it, which, in turn, generate reflected and refracted waves. In this case, generation of radiation is maximum in a direction perpendicular to the direction of oscillations of the charged particles and is minimum (absent) along the direction of the oscillations.

Electromagnetic waves

**Equipment:**
- A big optical bench with a protractor;
- a laser (red with two built-in polarizers) is fixed to a stand with a rubber band. For continuous operation, a turn-on button of the laser is clamped by a ring-shaped fixer. **Laser radiation is dangerous to the eyes! Do not direct its radiation towards the eyes!**
- a laser (green) with one polarizer;
- a polarizer with a protractor;
- a measuring device consisting of a photodiode with a holder and a multimeter in a ammeter mode for DC measurements;
- a glass plate (a refractive index of glass $n = 1.54$);

XXIV International olympiad "Tuymaada". Physics.
Senior league
Problem «Pyrometry»

Estimation of errors in this work is not required!

Theory. Necessary information

A theoretical formula of a spectral intensity of radiation (i.e. intensity per unit wavelength) of a heated body surface versus wavelength and temperature is given by Planck’s law with account of spectral emissivity coefficient ε(λ):

$$E_o(T, \lambda) = \varepsilon(\lambda) \cdot \frac{2 \pi h c^2}{\lambda^5} \cdot \frac{1}{\exp\left(\frac{hc}{kT\lambda}\right) - 1} \text{ (W/m}^2\text{)},$$

(1)

where $h = 6.626 \times 10^{-34}$ J·s is the Planck constant, $c = 3 \times 10^8$ m/s is the speed of light, $\lambda$ is the radiation wavelength, $T$ is the surface temperature in Kelvin.

Within a visible spectral range for moderate temperatures (below 5,000 K) $hc/kT\lambda \gg 1$, so “1” in a denominator of formula (1) can be neglected (so called Wien’s approximation).

In this work, it is required to study an emission spectrum of an incandescent lamp and in accordance with the results of the measurements, using Planck’s formula in Wien’s approximation, to estimate an absolute temperature of a filament of the lamp. In doing so consider that the emissivity $\varepsilon$ of the filament does not depend on the radiation wavelength $\varepsilon(\lambda) = \text{const}$ (an approximation of the so-called "gray" body).

Equipment

The lamp in a holder with a power source; two lenses in holders; a diffraction grating in a holder (a grating period $d = 1 \mu$m); a photodiode in a holder is covered by a white screen with a slit diaphragm; a rotary optical bench with a protractor, screen, multimeter, ruler, small sheet of white paper, napkin, adhesive tape (one-sided and double-sided), scissors (on demand).
The lamp and photodiode are fixed to a magnetic strip, which can be moved (up-down/left-right) relative to the holder during the measurements and adjustment of the system. The photodiode is fixed to strip rigidly (glued); the lamp is tightly clamped in the strip, but, if necessary, its position relative to the strip can be changed by rotating, for example, around its axis (keep in mind that during operation the lamp can become very hot).

To conduct the measurements with the photodiode, it should be connected to the multimeter in a DC voltmeter mode. A current of the photodiode for a given narrow spectral emission band $\lambda \pm \Delta \lambda$ is proportional to an intensity of incident light, so we will measure the light intensity in units of voltage (V).

**Task**

1. Assemble an experimental setup for conducting spectral studies (a spectrometer). Describe in detail your spectrometer design, an adjustment procedure and a principle of its operation. Draw a diagram of the spectrometer specifying its main parameters.
2. Describe a method of measurements of the emission spectrum $E(\lambda)$ of the incandescent lamp using your spectrometer.
3. Experimentally study the emission spectrum of the incandescent lamp $E(\lambda)$ in a working interval of the photodiode. A graph of the spectral sensitivity of the photodiode is shown in a figure.
4. Write Planck's formula in Wien's approximation.
5. Compare an experimental emission spectrum with a theoretical one (with Planck's formula in Wien's approximation).
6. Based on the results of measurements, estimate the absolute temperature of the filament of the lamp.

**Attention! During the work, a bulb and lamp base will become very hot, do not grasp them with your bare hands.**

**Junior league**

**Problem «Polarizer»**

**Solution**

**Item 1.**

1.1. A laser has two polarizers. At output from the first polarizer we will have radiation, in which a vector $\hat{E}_1$ oscillates in a certain plane. The second polarizer (analyzer) will pass through itself a strength component $E_2 = E_1 \cos \varphi$, where $E_1$ is the field strength after the output from the first polarizer (Fig. 1.1), $\varphi$ is an angle between planes of transmission of the polarizers.
Fig. 1.1.
Since an intensity of light is proportional to the square of an amplitude of the strength of a light wave coming out of the analyzer, then
\[ I_2 = I_1 \cos^2 \varphi \]  
(1.1).

1.2. To measure dependence of radiation intensity at the output from the laser on the angle of rotation of the analyzer \( I(\varphi) \), we assemble a layout shown in Fig. 1.2. To measure the angle, it is necessary to make an angular scale on the laser with the help of graph paper and adhesive tape.

Fig. 1.2. Layout to measure radiation intensity. L is laser, FD is photodiode, A is ammeter.

1.3. Turning the analyzer with the scale, we determine dependence of current of the photodiode on the angle of rotation;
1.4. We plot graphical dependence of the photocurrent (the light intensity) on the square of cosine (Fig. 1.3). It is also considered correct to plot dependence of a ratio of actual current to the maximum current on \( \cos^2 \varphi \).

Fig. 1.3
From data in Fig. 1.3 it follows that \( I_2 = I_1 \cos^2 \varphi \), corresponding to theoretical formula (1.1).

Item 2. Determination of direction of polarization of polarizer with protractor \( \varphi_{pol} \).
2.1. When particles oscillate, the radiation along the direction of their oscillations does not occur. In this case, the oscillation of the electric field strength vector \( \vec{E} \) will lie in a plane of incidence (the plane containing rays of incidence, reflection and normal to the surface) (Fig. 2.1). At the same time the intensity of a reflected beam is minimal, when an angle between reflected and refracted rays is equal to \( 90^\circ \) (Fig. 2.1). A minimum of lighting will be observed on a screen.
2.2. In accordance with Snell's law:
\[
\frac{\sin \alpha}{\sin \beta} = \frac{n_2}{n_1} = n_2
\]
(2.1)
where \(n_1 = 1\) is the refractive index of air, \(n_2\) is the refractive index of a plate (Fig. 2.2).
If the angle between the reflected and refracted rays is equal to 90° and \(\beta=90-\alpha\), then expression (2.1) takes the form:
\[
\frac{\sin \alpha}{\sin \beta} = \frac{\sin \alpha}{\sin(90-\alpha)} = \frac{\sin \alpha}{\cos \alpha} = \tan \alpha = n_2 \rightarrow n_2 = \arctan \alpha
\]
(2.2)
For a given film (glass) \(\alpha=57° \rightarrow n=1.54\).
2.3. It is recommended to orient the laser at the maximum of the output radiation. Instead of the measuring device we install a glass plate and direct the reflected light to the screen (Fig. 2.3). The glass is rotated through an angle of about 57° from the normal. If the intensity of light on the screen increases, when the plate is turned both toward smaller and larger angles, then the plane of polarization of the polarizer will correspond to the plane of incidence. We write the angle \(\phi_{pol}\) on the polarizer scale, corresponding to the horizontal plane. This value will correspond to the direction of transmission of the polarizer.

2.3 Determination of absorption coefficient of given polarizer.
The transmission planes of the polarizers of the laser and an external polarizer are oriented towards the transmission maximum (we determine it, using the measuring device). Rotating the laser analyzer, we determine the dependence of the photocurrent on the angle of rotation. The intensity of light will be determined by an expression: \(J_2 = T J_1 \cos^2 \varphi\), where \(T\) is a transmission coefficient. After plotting the dependence of the current \(J_2\) on \(\cos^2 \varphi\), we find an angular coefficient \(a_2\) (Fig. 2.4). After calculating a ratio of the found coefficients to the values measured in Item 1 without the polarizer, according to a formula \(k=1-a_2/a_1\), we find the absorption coefficient \(k\).

\[
k=1-0.67/5.84=0.89
\]
**Fig. 2.4. Dependence of current on angle of rotation of analyzer**

\[
y = 0.67x - 0.03
\]

Item 3. To determine the refractive index of an opaque plate, we place the test plate in place of the glass plate. At the same time, positions of the polarizer and the laser should not change. We turn the plate until we achieve the minimum intensity of lighting on the screen, which corresponds to 55°. Next, using formula (2.2), we determine the refractive index of the plate \( n = 1.43 \).

Item 4. Determination of degree of polarization of radiation of green laser. Rotating the polarizer on the laser, we measure the dependence of the current on the photodiode on the angle of rotation of the polarizer. For measurements in the same way as in Item 1, the angular scale must be made. From the graph of the dependence of the photocurrent on the angle of rotation of the polarizer in polar coordinates, we find \( I_{\text{max}} \) and \( I_{\text{min}} \), and we find the degree of polarization of the laser radiation using formula (1).

**Evaluation criteria**

<table>
<thead>
<tr>
<th>No.</th>
<th>Task</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>A device was assembled to measure dependence of radiation intensity at output of a red laser on the angle of rotation of the analyzer ( \varphi ). For this purpose, an angular scale is made on the analyzer.</td>
</tr>
<tr>
<td>1.2</td>
<td>Experimental data were obtained on dependence of photodiode current on the angle of rotation (at least 5 points, 0.2 per point, at least 3 within limits 90°).</td>
</tr>
<tr>
<td>1.3</td>
<td>Dependence of photocurrent on ( \cos^2 \varphi ).</td>
</tr>
<tr>
<td>1.4</td>
<td>A good agreement with a theoretical formula ( J_2 = J_1 \cos^2 \varphi ).</td>
</tr>
<tr>
<td>2.1</td>
<td>Snell's law.</td>
</tr>
<tr>
<td>2.2</td>
<td>It was indicated that an angle between reflected and refracted rays is equal to ( \pi/2 ).</td>
</tr>
<tr>
<td>2.3</td>
<td>There were obtained a formula ( \arctg \alpha \equiv n ) and an angle 57° is found.</td>
</tr>
<tr>
<td>2.4</td>
<td>A polarization angle of the polarizer was correctly found.</td>
</tr>
<tr>
<td>2.5</td>
<td>Experimental data obtained for determination of an absorption coefficient in a range from 0° to 90° and more (at least 5 points, 0.2 per point, but not less than 3 points).</td>
</tr>
<tr>
<td>2.6</td>
<td>A transmission coefficient was obtained on the basis of data of Items 1-2.</td>
</tr>
</tbody>
</table>
2.7 A value of the absorption coefficient was obtained:
with an accuracy to 10%,
with an accuracy to 20%.
The absorption coefficient was derived on the basis of
current measurement with one direction of transmission
of the polarizer.

A value of the refractive index of the plate was found
with an accuracy to 5%.
The value of the refractive index of the plate was found
with the accuracy to 10%.

There were obtained experimental dependences of the
photocurrent on the angle of rotation of the polarizer.
A graph of \( I(\phi) \) was plotted in polar coordinates.
The dependence was plotted in Cartesian coordinates.
Values \( I_{\text{max}} \) and \( I_{\text{min}} \) were found graphically.
A correct value of the degree of polarization of
radiation of green laser was obtained.

Total -20

Senior league
Problem «Pyrometry»
Solution

Principal scheme and adjustment of spectrometer.

Principal scheme of spectrometer
A scheme of a device for performing spectral measurements
(spectrometer) is shown in Fig. 1.
A light source (lamp) is placed in focus of the first lens, so that a parallel
beam of light is incident on a diffraction grating and, after the
diffraction, is collected in focus of the second lens on a photodiode. A
screen is set to prevent exposure of the photodiode to rays from the source.

Fig.1 General scheme of device

Adjustment of spectrometer.
This system is adjusted as follows: we set “zero” on a degree scale of a
protractor and, by small displacements of the lamp, photodiode, and
lenses we ensure that the zeroth (uncolored) diffraction maximum falls
exactly on a slit diaphragm of the photodiode. At the same time, a clear
image of a lamp spiral is formed on the photodiode. We place the lamp
spiral in a vertical position, turning the lamp in a holder around its axis.


Wavelength \( \lambda \)
We work with the first diffraction maximum. A wavelength is
determined using a formula:
\[
\lambda = \frac{d \sin \phi}{m}
\]
where \( d = 1 \, \text{\mu m} \), \( \phi \) is an angle determining a direction to the first
diffraction maximum. The angle \( \phi \) is read on the scale of the protractor.
Background
In order to take into account background illumination, we will block by a piece of paper a beam of light diffracting on the grating. Background voltage on the photodiode $U_0 = 6.9$ mV. When studying an emission spectrum, we will subtract this value from voltmeter readings $U$.

Taking into account relative spectral sensitivity. Working formula
To take into account differing sensitivity of the photodiode to different wavelengths, we introduce a correction factor using a graph $S(\lambda)$. As a result, a working formula for determining an intensity $E(\lambda)$ of radiation at a given wavelength in a fixed range $\lambda \pm \Delta \lambda$ looks like this:

$$E(\lambda) = \frac{(U - U_0)}{S(\lambda)}$$

3. Radiation spectrum. Experiment
Results of measurements and calculation of the emission spectrum $E(\lambda)$ of the lamp in the working region of the photodiode $\lambda = 420 - 680$ nm are presented in Table 1. A value of the relative spectral sensitivity corresponding to the wavelength $\lambda$ is indicated in a column with $S(\lambda)$. A value of $S(\lambda)$ was determined according to a graph presented in the Problem Statement.

<table>
<thead>
<tr>
<th>$\varphi$ (degree)</th>
<th>$\lambda$(nm)</th>
<th>$1/\lambda$(nm$^{-1}$)</th>
<th>$U$(mV)</th>
<th>$S(\lambda)$</th>
<th>$E(\lambda) = \frac{(U - U_0)}{S(\lambda)}$(mV)</th>
<th>$\ln(\lambda^2E(\lambda))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>43</td>
<td>682</td>
<td>0.001466</td>
<td>180</td>
<td>0.4</td>
<td>432.75</td>
<td>38.69531</td>
</tr>
<tr>
<td>42</td>
<td>669</td>
<td>0.001495</td>
<td>190</td>
<td>0.65</td>
<td>295.5358</td>
<td>38.21772</td>
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<tr>
<td>41</td>
<td>656</td>
<td>0.001524</td>
<td>212</td>
<td>0.69</td>
<td>297.2464</td>
<td>38.12537</td>
</tr>
<tr>
<td>40</td>
<td>643</td>
<td>0.001555</td>
<td>227</td>
<td>0.72</td>
<td>305.6944</td>
<td>38.05331</td>
</tr>
<tr>
<td>39</td>
<td>629</td>
<td>0.001599</td>
<td>242</td>
<td>0.85</td>
<td>276.5882</td>
<td>37.84319</td>
</tr>
<tr>
<td>38</td>
<td>616</td>
<td>0.001623</td>
<td>248</td>
<td>0.9</td>
<td>267.8889</td>
<td>37.76081</td>
</tr>
<tr>
<td>37</td>
<td>602</td>
<td>0.001661</td>
<td>245</td>
<td>0.9</td>
<td>264.5556</td>
<td>37.57934</td>
</tr>
<tr>
<td>36</td>
<td>588</td>
<td>0.001704</td>
<td>243</td>
<td>0.98</td>
<td>240.9184</td>
<td>37.36809</td>
</tr>
<tr>
<td>35</td>
<td>574</td>
<td>0.001742</td>
<td>230</td>
<td>0.98</td>
<td>227.6531</td>
<td>37.19097</td>
</tr>
<tr>
<td>34</td>
<td>559</td>
<td>0.001789</td>
<td>214</td>
<td>1</td>
<td>207.13</td>
<td>36.96395</td>
</tr>
<tr>
<td>33</td>
<td>545</td>
<td>0.001835</td>
<td>194.5</td>
<td>0.97</td>
<td>193.4021</td>
<td>36.7687</td>
</tr>
</tbody>
</table>

Table 1. Emission spectrum of radiation of incandescent lamp $E(\lambda)$

1. Planck's formula in Wien's approximation.
In the studied range of wavelengths, an exponential function value (assuming that $T < 5000$ K) is much greater than unity:

$$\exp\left(\frac{hc}{\lambda kT}\right) >> 1$$

Neglecting “1” in a denominator in comparison with the exponential, we obtain Planck's formula in Wien's approximation:

$$E(T, \lambda) = \frac{2 \pi h c^2}{\lambda^5} \cdot \frac{\exp\left(\frac{hc}{\lambda kT}\right)}{\lambda^5}$$

2. Comparison of theory with experiment
It follows from the theoretical formula obtained in Item 3 that when $\varepsilon(\lambda) =$ const:

$$\ln \lambda^2E(\lambda) = -\frac{hc}{kT} + \text{const} .$$

To compare an experiment with a theory, we present the experimental data in a form of a graph, plotting values of $x = 1/\lambda$ on the X-axis, and values of $y = \ln \lambda^2E(\lambda)$ on the Y-axis. In such coordinates, the graph of theoretical dependence has the form of a straight line $y = \beta x + \text{const}$, a slope of which is $\beta = \frac{hc}{kT}$. The graph of the dependence of $E(\lambda)$ in these linearized coordinates is shown in Fig. 2. It can be seen that the experimental points fall well on the straight line $y = -5034x + 45.9$. The slope of the straight line:

$$\beta = \frac{hc}{kT} = -5034 \text{ nm}.$$
1. **Temperature of filament**
   We calculate an absolute temperature of a filament using a formula:
   \[ T = -\frac{h c}{k \theta} \approx 2860 \text{ K} \]
   *(the obtained value of temperature completely justifies our Wien's approximation of Planck's formula)*

**Evaluation criteria**

<table>
<thead>
<tr>
<th>№</th>
<th>Task</th>
<th>max</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td><strong>Experimental setup</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.1 A general view of a basic layout of the spectrometer: a lamp, a lens, a diffraction grating, another lens, a photodiode, a screen to prevent light of the lamp from directly falling into the photodiode.</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>1.2 A correct adjustment of the spectrometer is carried out: a filament of the lamp is vertical (1) and in focus of the first lens (1), the photodiode is in focus of the second lens (1), an adjustment of this system is according to the zeroth maximum ( (\varphi = 0) ) (1)</td>
<td>4</td>
</tr>
</tbody>
</table>